FBP'97

BOOK OF ABSTRACTS

FREE BOUNDARY PROBLEMS, THEORY AND APPLICATIONS

Herakleion, Crete, Greece
8–14 June, 1997

The University of Crete

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FREE BOUNDARY PROBLEMS, THEORY AND APPLICATIONS

Herakleion, Crete, Greece
8–14 June, 1997

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H. W. Alt

Universität Bonn, Germany

Entropy in non-equilibrium thermodynamics

We give an answer to the question whether the entropy principle in rational thermodynamics can be derived from more basic principles. We formulate an "evolution principle" and show that in certain classical situations it is equivalent to the entropy principle. We then discuss cases related to order parameters and interfaces.

Alexis Bonnet

Paris, France

On the number of singular points on a free-boundary

On two particular problems, I will describe how one can get informations on the number of pieces and singular points of a free-boundary. The first problem comes from the minimization of the Mumford-Shah functional in computer vision. The second set of results, which have been derived by Regis Monneau (CERMICS, PARIS), deals with the obstacle problem in two dimensions.

Andrejs Cebers

Institute of Physics, Latvia

2D Free surface flows of magnetic liquids

Competing long-range magnetic repulsion and surface tension forces are causing the development of remarkable labyrinthine patterns in Hele-Shaw cells. The control parameter (magnetic Bond number) of those pattern formation phenomena is determined by the ratio of the self-magnetic field and capillary forces. In the frame of the simple model accounting for the self-magnetic field energy of magnetic liquid in Hele-Shaw cell and surface energy of free boundaries the threshold values of the magnetic Bond number for various instability modes in different geometries (circular droplet, infinite stripe etc.) have been calculated. The patterns developing at supercritical values of the magnetic Bond number are determined by the numerical solution of the free boundary problem for the magnetic liquid motion in Hele-Shaw cell under the action of magnetic and capillary forces. Here the corresponding numerical results obtained for the magnetic liquid Hele-Shaw flows by applying boundary integral equation technique are given. Numerical simulation of the pattern formation in simplest case of enough small magnetic Bond numbers illustrate the establishment of nonsymmetric figures of equilibrium (dumbbell, three lobe figure). It is shown by numerical simulation results that development of the figures of equilibrium with the symmetry axis of the higher order than three is impossible due to vertex splitting instability. No peculiarities in the energy time dependence curves are noticed at vertex splitting events. It is shown by numerical simulation that after several vertex splitting events at intermediate stages the long-living transient configuration close to Steiner trees are formed. Under longer time periods evolution of those configurations to the simpler tree-like structures takes place. That evolution opposite to the first stage is driven by the surface tension forces. The influence of the viscosity contrast on those phenomena is studied by the comparison with the case of the labyrinthine instability development of the bubble in infinite plane layer of magnetic liquid.

As it is possible to conclude from the numerical simulation results of the labyrinthine instability of the magnetic liquid drops for the patterns formed stripes of the constant width are characteristic.

From the analysis of the properties of the stripe structures several very interesting conclusions can be drawn:

1) in the case of the infinite system of the straight magnetic stripes in the Hele-Shaw cell effective surface tension of the equilibrium pattern due to the long-range magnetic forces is exactly equal to zero;

2) at enough fast magnetic field increase undulation instability of the stripe system leading to the formation of chevron patterns exists;

3) the similar concept of the negative effective surface tension can be applied as well to the description of the disordered systems of the magnetic liquid stripes, for example, magnetic liquid foams;

4) various defects in the magnetic liquid stripe systems can be described on the basis of 2D smectic approach.

Several theoretical conclusions mentioned above are illustrated by the numerical simulation of the corresponding free boundary problems. So for magnetic liquid foam structure at the magnetic Bond number increase the undulation instability of the single stripes is observed. It is illustrated also by the numerical simulation of the single stripe that the development of the overextensions leading to the rupture of the stripe is impossible due to the vertex splitting instability as the result the formation of the alternating finger patterns.is observed.

Finally at the end of the first part several further issues of the labyrinthine pattern formation are considered:

a) topological instabilities at the dumbbell relaxation at switching off the magnetic field and displacement of the stripe;

b) peculiarities of the influence of the long-range magnetic interactions on Rayleigh-Taylor and Saffman-Taylor instabilities.

In the second part some results for the 2D free boundary Stokes flows of the magnetic liquid in ac fields are given. As it is known from theoretical investigations for 2D magnetic fluid droplet the instability with respect to the formation of the star fishes exists. By numerical simulation of the corresponding free boundary problem it is shown that 2-spike shape has the lowest energy and is usually obtained at the relaxation from arbitrary initial shape. 3- spike shape can exist as metastable state. Shapes with higher number of the spikes are impossible due to the vertex splitting instability. According to the numerical simulation results different regimes of the magnetic liquid droplet motion are possible in low frequency rotating magnetic fields. At low frequencies the droplet is elongated and follows synchronously the field. Behaviour at higher frequencies depends on the value of magnetic Bond number - for magnetic Bond number greater as critical one at high frequencies complex regimes of the droplet motion including forward and backward rotations arise. Simple analytical model for the droplet behaviour in rotating field can be derived which shows good agreement with the numerical simulation data. With that model devils staircase for the droplet rotation frequency in dependence on elliptically polarized field rotation frequency is obtained.

Joint work with **I.Drikis and S.Lacis**.

Richard Ewing

Texas A & M University, USA

FREE BOUNDARY PROBLEMS ARISING IN APPLICATIONS OF FLUID FLOW IN POROUS MEDIA

Understanding the fate and transport of contaminants to determine water quality and to develop remediation strategies or optimizing the recovery of hydrocarbons in petroleum applications each require the ability to model multiphase flow in heterogeneous three-dimensional reservoirs. Model equations and corresponding parameters must be determined at the appropriate length scales to describe the scaled physics of flow. The interaction of the various phases in multiphase

flows yields very sharp fluid interfaces that travel accross the reservoir under the influence of injection and production rates. The speed of these fronts depends in a highly nonlinear way on the dynamic state of the fluids and fluid pressures. These free boundary problems must be treated computationally due to the strong nonlinearities and couplings. Effective simulators require accurate numerical methods on general geometries. Use of mixed finite element methods and local grid refinement will be discussed. Example calculations for field simulations in aquifers or reservoirs with complex boundaries will be presented. Parallelization of the codes will also be discussed.

Antonio Fasano

Universita di Firenze, Italy

TWO SCALE DIFFUSION PROBLEMS

In many problems of practical interest diffusion of mass and/or heat occurs in a medium having a fine component in which the same process takes place, possibly accompained by other phenomenon, usually coupled with the bulk process. A remarkable example is a heat conducting medium undergoing nucleation of crystals, in which the whole crystallization process is crucially influenced by temperature and is accompained by the release of latent heat. Another interesting case is the freezing of droplets of liquid dispensed in another liquid with lower melting temperature. The droplets phase change effects the heat diffusion process on the large scale and in turn the solidification of each droplet is governed by the local value of the bulk temperature. Filtration through porous media can also provide peculiar examples of two-scale diffusion with free boundaries, if the solid matrix contain "active" components (e.g. swelling moisture absorbing grains). An extremely complex process exhibiting two-scale diffusion is polymerization of gaseous monomers in an agglomerate of ting catalytic particles. Here we have to consider the monomer diffusion in the agglomerate (the large scale), and through the polymer shell growing around the particles (the small scale), the latter being a free boundary problem. Not only it is important to consider the simultaneous heat diffusion (polymerization is highy exothermic), but also the expansion velocity fields in the two scales must be reconstructed. A review of mathematical models will be presented.

Carlos Kenig

University of Chicago, USA

MONOTONICITY FORMULAS AND REGULARITY OF TWO PHASE FREE BOUNDARY PROBLEMS

We will discuss the role of monotonicity formulas in the proof of regularity of free boundaries in two phase situations. We will emphasize some recent works, on parabolic problems (joint with L.Caffarelli),and on elliptic problems of Poisson type (joint with L.Caffarelli and D.Jerison).The results have applications to combustion theory (in the first case) and to fluid flows (in the second case).

Masayasu Mimura

The Univeristy of Tokyo, Japan

During the past decade, various interesting phenomena of patterns and interfaces have been observed in chemical reaction processes, solidifications, combustion, binary alloy, liquid crystals in the fields of natural sciences and engineering. In order to understand the dynamics of patterns, most of them are described by certain types of free boundary problems. Along this line, I would like to demonstrate several biological pattern formation arising segregation or aggregation of biological individuals, which are described by interfacial or free boundary problems. The tool used there is a singular limit analysis.

Stanley Osher

University of California Los Angeles, USA

THE LEVEL SET METHOD FOR ANALYZING AND COMPUTING FREE BOUNDARY PROBLEMS

In 1987, together with J. A. Sethian, we devised a new numerical procedure for capturing the motion of fronts. The method uses a fixed grid and finds the front as a particular level set, moving with time, of a scalar function. The technique has since been applied to an enormous number of topics, ranging from computer vision to computational fluid dynamics and beyond. The method handles topological merging and breaking, works in any number of space dimensions, does not require that the moving front be written as a function, captures steep gradients and cusps in the front (using modern capturing schemes, e.g., ENO) and is easy to program. Theoretical justification, involving the notion of viscosity solutions, has been given. Many applications and extensions have been recently developed. For this talk we will describe the method applied to multiphase compressible interface motion, Stefan problems, and problems in materials science. Additionally, the level set point of view enables us to obtain some new analytic results concerning crystal growth and shape.

Constantine Pozrikidis

University of California at San Diego, USA

DYNAMIC SIMULATIONS OF THE FLOW OF SUSPENSIONS OF LIQUID DROPS

Suspensions of liquid drops are encountered in a variety of chemical engineering and bioengineering systems, and have been studied using a host of theoretical and experimental methods. In this research, we carry out dynamic simulations of the flow of suspensions of two-dimensional liquid drops with constant interfacial tension in simple shear flow. The computations rely on the method of interfacial dynamics, which is an advanced implementation of the Boundary Integral Method. The numerical task is the iterative solution of a system of second kind integral equations. The convergence is expedited by deflating the spectrum of the double-layer operator. The results allow us to characterize the motion in terms of effective rheological properties, and also study the dynamics of the evolving microstructure. The pair distribution function for suspensions of moderate-viscosity drops is shown to be significantly different from that of suspensions of rigid particles, and this is explained in terms of the ability of the drops to slide over each other during collision. The seemingly random motion of one individual drop in the suspension is described in terms of an effective hydrodynamic self-diffusivity which is computed from the results of the simulations.

Mike Savage

University of Leeds, UK

MENISCUS ROLL COATING AND THE BEAD BREAK INSTABILITY

Free boundary problems are ubiquitous in liquid film coating. In this presentation attention is focused on Meniscus Roll Coating - a coating regime which arises when the inlet is starved so that flow rate is small and a "fluid bead" is located in the nip bounded by two moving rolls and two free surfaces. The pressure field is entirely subambient dominated by capillary pressure at the upstream meniscus. The flow consists of a fluid transfer jet or "snake" circumnavigating two large, closed recirculations in order to transport inlet flux to the upper roll. The stability of the two - dimensional base flow to small amplitude perturbations is considered. It is shown that , as upper roll speed is gradually increased, the upstream meniscus passes through the nip and instability in the form of bead break soon follows.

Jose Alexandre Scheinkman

University of Chicago, USA

SOME FREE BOUNDAY PROBLEMS THAT ARISE IN ECONOMICS AND FINANCE

In this lecture we will discuss two examples of free boundary problems arising in economics. The first example arises in the theory of asset pricing in financial markets. The second example deals with dynamic optimization in the presence of irreversible decisions, and is inspired by questions on the economics of environmental management.

H.A. Stone

Harvard University, USA

FREE-BOUNDARY PROBLEMS IN FLUID DYNAMICS WITH EMPHASIS ON VISCOUS FLOWS

An overview of the wide variety of free-boundary problems that arise in fluid dynamics will be given. Flows for which integral equation methods (for either high or low Reynolds numbers), numerical approaches valid for arbitrary Reynolds numbers, one-dimensional approximations, and similarity solutions are useful will be summarized and their physical applications indicated. Also, cases where usually distinct analytical and numerical approaches are combined (e.g. integral equation methods with the lubrication approximation) will be demonstrated. Several examples drawn from our own research (and also, of course, the research of others) in viscous, multiphase flows will illustrate the interplay of numerics and experiments for obtaining additional physical insight, including coalescence of deformable drops in dilute suspensions, drop and thread breakup, and the conical shape of the interface between two dielectric liquids exposed to an electric field.

FOCUS SESSION

FREE BOUNDARIES IN COMBUSTION

ORGANIZED BY

Claude-Michel Brauner

Université Bordeaux I , France

LIST OF SPEAKERS

Claude-Michel Brauner, Université Bordeaux I , France
Bill Dold, UMIST, UK
Amable Linan, Universidad Politecnica de Madrid, Spain

Claude-Michel Brauner

Université Bordeaux I, France

A SURVEY OF RECENT MATHEMATICAL RESULTS ON FBPS IN COMBUSTION

I will report on the following papers:

1) L.A Caffarelli (Courant Institute), C. Lederman (Buenos Aires) and N. Wolanski (Buenos Aires): *Pointwise and viscosity solutions for the limit of a two-phase parabolic singular perturbation problem*

2) C. Lederman and N. Wolanski (Buenos Aires): *Viscosity solutions and regularity of the free boundary for the limit of an elliptic two-phase singular perturbation problem*

3) C. Lederman (Buenos Aires), J. L Vazquez (Univ. Autonoma de Madrid) and N. Wolanski (Buenos Aires): *Uniqueness of solution to a free boundary problem from combustion*

These papers deal with models of propagation of premixed flames of the following types,

$$\Delta u^\varepsilon - u_t^\varepsilon = \beta_\varepsilon(u^\varepsilon) \quad \text{or} \quad \Delta u^\varepsilon = \beta_\varepsilon(u^\varepsilon),$$

where $1/\varepsilon$ stands for the large activation energy and u^ε is a normalized temperature, $\beta_\varepsilon(s) = \frac{1}{\varepsilon}\beta(\frac{s}{\varepsilon})$. Convergence results have been obtained as $\varepsilon \to 0$ to FBPs

$$\Delta u - u_t = 0 \quad \text{or} \quad \Delta u = 0,$$

with free boundary conditions such that $u = 0$, $u_\nu = \sqrt{2M}$ (one-phase problem) or $(u_\nu^+)^2 - (u_\nu^-)^2 = 2M$ (two-phase problem) where M is related to β.

In the two-phase parabolic case, limits are shown to be solutions to the FBP in pointwise and viscosity senses. In the elliptic case, among other results, two-phase nondegenerate limits are shown to be classical solutions. In the one-phase parabolic case, different notions of solution to the FBP are proved to coincide and produce a unique solution.

4) A. Bonnet (ENS Paris) and F. Hamel (Université Paris VI): *Existence of non planar solutions of a simple model of premixed Bunsen flames*

In a Bunsen burner with long shaped aperture the flame front has a bidimensional conical shape. Far away from the symmetry axis it behaves like a planar front with constant speed c_0. It is proved the existence of a solution to

$$\Delta u - cD_y u + f(u) = 0 \quad \text{in } R^2,$$

with two different limits as the radii go to infinity: $u \to 0$ as $r \to \infty$ in an open cone of aperture angle α determined by $c = c_0/\sin\alpha$, and $u \to 1$ in the open complementary cone.

5) J. Audounet (Université Toulouse 3), V. Giovangigli (Ecole Polytechnique) and J.-M. Roquejoffre (Université Toulouse 3): *An integral equation modelling the propagation of a point source initiated flame*

The time evolution of the radius R of a lean spherical flame, is described by the integro-differential equation (Joulin, 1985)

$$R\partial_{1/2}R = R\log R + Eq(t), \quad R(0) = 0,$$

where $\partial_{1/2}$ denotes the time derivative of order $1/2$. The function $q(t)$ represents the point source, and the parameter E represents its strength. It has been proved that the flame radius either tends to 0, or to 1, or to $+\infty$, according to whether the parameter E is below, equal to, or above a critical parameter $E_{cr}(q)$. Asymptotics of the different regimes have also been obtained.

6) G.S. Namah (Université Bordeaux 1) and J.-M. Roquejoffre (Université Toulouse 3): *A class of nonlinear parabolic and Hamilton-Jacobi equations arising in solid propellant combustion*

A qualitative model for the propagation of a flame front in a solid propellant reads as follows.

$$u_t - \Delta u = R(X,u)\sqrt{1+|Du|^2}, \quad X \in R^2.$$

It is proved that periodicity is space generates periodic in time solutions. Related Hamilton-Jacobi equations are also studied in the class of viscosity solutions. The corresponding homogenization problems (by letting the spatial period go to zero) have also been analyzed, to which effective ODE's are derived. For this end, profit is taken of the knowledge of the time-asymptotic behaviour of the above solutions.

7) C.M. Brauner (Université Bordeaux 1), A. Lunardi (Università di Parma) and Cl. Schmidt-Lainé (ENS Lyon): *Stability and bifurcation of non planar travelling fronts in a two-phase problem*

8) C.M. Brauner (Université Bordeaux 1), J. Hulshof (University Leiden) and Cl. Schmidt-Lainé (ENS Lyon): *The saddle point property for focusing selfsimilar solutions in a one phase problem*

In the limit of high activation energy, certain models consist of one- or two-phase problems such as

$$u_t = \Delta u + f(u, Du)$$

on both sides of a free boundary $x = \xi(y,t)$, where $u = u_*$ is given and jump conditions on the normal derivative of u are prescribed. An adequate splitting of the unknown yields the elimination of the front and leads to a fully nonlinear evolution equation. The latter formulation is specially efficient in the nonlinear stability analysis of travelling wave (TW) solutions.

In a model problem in dimension 2, the planar TW becomes unstable at a critical value u_*^c of u_*, at which an infinite number of bifurcated branches of nonplanar TWs accumulates.

Similarly, the free surface can be eliminated in the context of spherical flames for studying stable and unstable manifolds of focusing self-similar solutions.

Amable Linan

Universidad Politecnica de Madrid, Spain

FBP IN DIFFUSION CONTROLLED COMBUSTION

Bill Dold

UMIST, U.K.

NON-MONOTONIC CURVATURE-DEPENDENT PROPAGATION OF A FRONT

If an interface propagates at a speed $S(\kappa)$ that depends non-monotonically on the mean curvature κ then the shape of the interface experiences an anti-diffusive instability whenever the derivative $S'(\kappa)$ is negative—taking κ to be positive when concave in the direction of propagation. While negative diffusion has connotations of non-physical behaviour, such as violating the second-law of thermodynamics, ill-posedness, etc., it turns out that if $S'(\kappa)$ is negative only in bounded windows of curvature-space then any growth that arises from the instability is nonlinearly bounded. Having a "diffusion coefficient" that changes sign, as κ changes, makes the problem more tractable than a problem with uniformly negative diffusion.

A great deal can be deduced about the appearance and behaviour of the instability, including the surprising result that, under very simple conditions, an interface can propagate with a negatively diffusive instability for an infinite time and remain well-posed with respect to initial conditions! The effect of noise is another matter altogether, and any noise that decays more slowly than a Gaussian spectrum will cause an instant anti-diffusive growth as soon as $S'(\kappa)$ becomes negative (that is, as normal propagation of the interface causes the value of κ to move through some threshold value). The full growth of the instability is still limited within the windows of anti-diffusive curvature.

Wide classes of steadily propagating solutions can also be found with piecewise continuous curvature. These resemble a phase-separation in curvature space into positively and negatively curved solutions. For appropriate models of $S(\kappa)$, normal propagation has the effect of driving solutions into the anti-diffusive regime where, seeded by noise, such phase separation would take place.

A simple generalisation of the Kuramoto-Sivashinsky equation should lead to very similar dynamics, with the added presence of a regularising term involving a fourth-derivative, or Laplacian of curvature. It is not clear at this stage how important the details of any such regularisation are likely to be in determining the dynamics. Unlike models for separation into physical phases (as opposed to phases of curvature) it is likely that regularisation would play a secondary role which does not change the overall dynamics very much.

An interesting and rich dynamic can be expected for fronts which propagate with fairly simple models for $S(\kappa)$. Models of this nature have been predicted using slowly, but not too slowly, varying flame analyses of mixtures near stoichiometry, reacting via one-step Arrhenius kinetics at large activation temperature with certain combinations of non-unit Lewis numbers. Calculations using fuller models of flames at not too weak curvatures or stretch-rates are needed to identify if the phenomenon can be found more generally. Because of the instability, experiments in regimes of negative $S'(\kappa)$ are likely to become problematic.

It is worth pointing out that not only flames can propagate at non-monotonic speeds $S(\kappa)$. Other forms of interface possess this kind of law, including the meandering of rivers. Studying the dynamics of this new class of problem promises to reveal features that will help in understanding the unsteady behaviour of a range of interfaces in which anti-diffusive instabilities are restricted to windows of solution space.

FOCUS SESSION

FREE BOUNDARIES IN SOLID MECHANICS

ORGANIZED BY

Michel Fremond

Ponts et Chaussees, France

LIST OF SPEAKERS

R. Luciano, Rome Univ. II, Italy
Pavel Krejčí, Weierstrass Institute for Applied Analysis and Stochastics, Germany
Martin Brokate, University of Kiel, Germany

R. Luciano
Rome Univ. II, Italy

Pavel Krejčí
Weierstrass Institute for Applied Analysis
and Stochastics, Germany

Hysteresis in thermoplasticity

We propose a model for the behavior of elastoplastic materials, where the material characteristics (elasticity modulus, yield surface) depend on temperature. The construction is based on the idea of Prandtl (1928) and Ishlinskii (1944) to describe real elastoplastic materials by a parallel combination of simple elastic – perfectly plastic elements parametrized by the yield thresholds with a distribution function which is to be identified from experiments. Rather than large systems of evolution variational inequalities, it is more convenient to work with their solution operators which belong to the class of hysteresis operators. In our thermoelastoplasticity model, we simply assume that the Prandtl-Ishlinskii distribution function depends also on temperature. Since the constitutive law is given in terms of hysteresis operators, it is natural to expect that also the internal energy and entropy ensuring the thermodynamical consistency of the model will have the form of hysteresis operators. Our main goal is to prove the well-posedness of the corresponding system of the equation of motion and energy balance equation. While the modeling part is independent of the space dimension, existence and uniqueness results for systems of partial differential equations involving temperature-dependent hysteresis operators are obtained only in one space dimension due to restrictions imposed by the Sobolev embedding theorems.

Joint work with **J. Sprekels**.

Martin Brokate
University of Kiel, Germany

Nonlinear kinematic hardening

We discuss constitutive laws in elastoplasticity of kinematic nonlinear hardening type; 'nonlinear' here means that the direction of the movement of the yield surface differs from the direction of the plastic strain increment by a nonlinear correction term. A representative model goes back to Armstrong and Frederick; its multisurface version, the Chaboche model, is widely used in engineering. We present results concerning the wellposedness and asymptotic behaviour of such models.

FOCUS SESSION

FREE BOUNDARIES IN MATERIAL SCIENCES

ORGANIZED BY

K.H.Hoffmann

Technische Universität München, Germany

LIST OF SPEAKERS

Vasilios Alexiades, Univ. of Tennessee, USA
Harald Haller, Technische Universität München, Germany
Tomas Roubicek, Charles University, Czech Republic
Barbara Wagner, Technische Universität München, Germany

Vasilios Alexiades

University of Tennessee, USA

ALLOY SOLIDIFICATION WITH CONVECTION IN THE MELT

Most models of alloy solidification are severely limited by the assumption of constant density, thus excluding all convective effects. We develop a thermodynamicaly consistent model for binary alloy solidification that incorporates energy, species and momentum conservation, constitutional supercooling, as well as temperature and concentration dependence of thermophysical parameters. The crucial aspect is the development of an equation of state capturing the thermochemistry of the phases. A numerical scheme will also be outlined.

Harald Haller

Technische Universität München, Germany

MICROMECHANICAL MODELS AND HOMOGENIZATION OF AN ALASTIC PLATE CONTAINING FIBERS OF SHAPE MEMORY ALLOYS

Based on a micromechanical approach and on a generalized Achenbach–Müller model for the shape memory alloys a thermomechanical model of an elastic thin plate containing fibers of shape memory alloys is derived. Existence and uniqueness of the solutions is proven.

Since the physical parameters (such as heat conductivity, elasticity coefficients, ...) are discontinuous and oscillate between different values characterizing each of the components, the numerical solution of the derived equations requires a lot of computation time and for many applications it is even not possible to solve the equations numerically.

Using homogenization techniques one can get a good approximation of the macroscopic behaviour of such heterogeneous materials by letting the parameter ϵ, which describes the fineness of the microscopic structure, tend to zero in the given equations and therefore the computation time can be reduced. We derive the homogenized (or effective) equations for the fiber–reinforced plate and prove the convergence of the solutions.

Numerical simulations of the amplitude control of the vibrating plate show a very good correspondence to experimental results.

Tomas Roubicek

Charles University, Czech Republic

TWINNING AND INELASTIC RESPONSE OF CRYSTALIC MATERIALS

Plasticity in crystalic materials can be caused beside slip also by twinning. This is basically caused by a dissipative martensitic phase-change created by thermal or dislocation mechanisms. These mechanisms are activated if the stress exceeds certain limit so that the potential barrier between particular phases became partly "transparent". In the scalar case, this problem seems to be modelled by the author's microstructure evolution model modified by considering the dissipation energy nonconstant, being activated by a sufficiently large stress. The phenomena like hysteresis, observed experimentally in shape-memory alloys exposed to time-varying loading, can thus be modelled at least for a simply twinned configuration.

Barbara Wagner
Technische Universität München, Germany

NUMERICAL AND ASYMPTOTIC RESULTS ON THE LINEAR STABILITY OF A THIN FILM SPREADING DOWN A SLOPE OF SMALL INCLINATION

For a thin liquid film moving down a slope, the lubrication approximation with a small Navier-Slip is used as a model. The travelling wave solution is derived for small inclination angle α, using singular perturbation methods, and compared to the numerical solution. For the linear stability analysis we combine numerical methods with the long-wave approximation and find a small but finite critical α^* below which the flow remains linearly stable. This is contrasted with the vanishing of the hump of the travelling wave solution. Finally, this prevailing of linear stability of the travelling wave at small inclination angles is compared with recent related results using a precursor model instead. Here though, a strong dependency on the magnitude of the contact angle is found, which, we think, has not been observed before.

FOCUS SESSION

SINGULARITIES OF INTERFACES, CUSPS, CAVITATION AND FRACTURE

ORGANIZED BY

Daniel D. Joseph

University of Minnesota, USA

LIST OF SPEAKERS

Daniel D. Joseph, University of Minnesota, USA
John Lowengrub, University of Minnesota, USA
Constantine Pozrikidis, University of California at San Diego, USA
Leonid Antanovskii, Monash University, Australia

Daniel D. Joseph

University of Minnesota, USA

CAVITATION AND THE STATE OF STRESS IN AFLOWING LIQUID

The problem of the inception of cavitation is formulated in terms of a comparison of the breaking strength or cavitation threshold at each point of liquid sample with principal stresses there. A cavity will open in the direction of the maximum tensile stresses which is 45^o from the plane of shearing in pure shear of a Newtonian fluid. A criterion of maximum tension is proposed which unifies the theory of cavitation, the theory of maximum tensile strengh of liquid filaments and the theory of fracture of amorphous solids. Experiments which support these ideas are discussed and some new experiments are proposed.

John Lowengrub

University of Minnesota, USA

CAHN-HILLIARD HYDRODYNAMICS AND TOPOLOGICAL TRANSITIONS

The competition between surface tension and instability often leads to singularities in interfacial flows in which interfaces collide and/or self-intersect and the topology of the flow changes. Before the singularity, sharp interface models provide good models of the flow. At the singularity time, however, these models break down and additional features of the flow must be considered such as the partial miscibility between "immiscible" fluid components. In this talk, I will introduce a new model (Cahn-Hilliard Hydrodynamics) in which sharp interfaces are replaced by microscopic mixing layers that account for the partial miscibility that real fluids always display. Analysis and simulations will demonstrate the model's ability to capture changes in topology.

Constantine Pozrikidis

University of California at San Diego, USA

NUMERICAL STUDIES OF SINGULARITY FORMATION AT FREE SURFACES AND FLUID INTERFACES IN TWO-DIMENSIONAL STOKES FLOW

We consider the analytic structure of interfaces in several families of steady and unsteady two-dimensional Stokes flows, focusing on the formation of corners and cusps. Previous experimental and theoretical studies have suggested that, without surface tension, the interfaces spontaneously develop such singular points. We investigate whether and how corners ad cusps actually develop in a time-dependent flow, and assess the stability of stationary cusped shapes predicted by previous authors. The motion of the interfaces is computed with high resolution using a boundary integral method for three families of flows. In the case of a bubble that is subjected to the family of straining flows devised by Antanovskii, we find that a stationary cusped shape is not likely to occur as the asymptotic limit of a transient deformation. Instead, the pointed ends of the bubble disintegrate in a process that is reminiscent of tip streaming. In the case of the flow due to an array of point-source dipoles immersed beneath a free surface, which is the periodic version of a flow proposed y Jeong & Moffatt, we find evidence that a cusped shape indeed arises as the result of a transient deformation. In the third part of the numerical study, we show that, under certain conditions, the free surface of a liquid film that is leveling under the action of gravity on horizontal or slightly inclined surface develops an evolving corner or cusp. In certain cases, the film engulfs

a small air bubble of ambient fluid to obtain a composite shape. The structure of a corner or a cusp in an unsteady flow does not have an unique shape, as it does at steady state. In all cases, a small amount of surface tension is able to prevent the formation of a singularity, but replacing the inviscid gas with a viscous liquid does not have a smoothing effect. The ability of the thin-film lubrication equation to produce mathematical singularities at the free surface of a leveling film is also discussed.

Leonid Antanovskii

Monash University, Australia

HYSTERESIAL BEHAVIOUR OF A POINTED DROP IN TAYLOR'S FOUR-ROLLER MILL

Time-dependent deformation of a two-dimensional inviscid drop subjected to quasi-steady straining flow, such as that created in Taylor's four-roll mill [Taylor, 1934] at slowly varying rotation rate of the cylinders, is addressed. Following the spirit of matched asymptotic expansions, the problem is decomposed into the outer Stokes problem with no drop effect, which is solved by the boundary-element method, and the inner free-boundary problem for the drop behaviour in unbounded fluid. Using complex variable technique, a broad class of explicit solutions is found, which are described by a rational conformal mapping of the unit disc onto the flow domain, with time-dependent coefficients satisfying ordinary differential equations. Some numerical simulations of the resulting equations are implemented, which suggest that, with increasing rotation rate of the cylinders, a sudden cusping of a sufficiently small drop occurs at a critical strain. This cusped configuration survives up to any strain, but when the strain is being gradually removed, the drop becomes suddenly rounded at a lower critical strain. This proves that, under certain circumstances, the behaviour of a 2D inviscid drop placed within a quasi-steady creeping flow is hysteresial.

FOCUS SESSION

NUMERICAL ANALYSIS OF FREE BOUNDARY PROBLEMS

ORGANIZED BY

Ricardo Nochetto
University of Maryland, USA

and

Claudio Verdi
Universitá di Milano, Italy

LIST OF SPEAKERS

Ralf Kornhuber, Universität Stuttgart, Germany
Maurizio Paolini, Universittá di Udine, Italy
Michael J. Shelley, New York University, USA
Alfred Schmidt, Univ. Freiburg, Germany
Pierre Gremaud, North Carolina State University, USA

Ralf Kornhuber

Universität Stuttgart, Germany

On the fast solution of free boundary problems

A wide range of free boundary problems occurring in engineering and industry can be rewritten as a minimization problem for a strictly convex, piecewise smooth but non–differentiable energy functional. The algebraic solution of the related discretized problem is a very delicate question, because usual Newton techniques can not be applied.

We propose a new approach based on convex minimization. Each iteration consists of one step of a globally convergent descent method and an additional energy reducing correction. While the initial (fine grid) smoothing provides global convergence of the overall iteration, the subsequent (coarse grid) correction is intended to accelerate the convergence speed. We present a general convergence theory involving asymptotic multigrid convergence rates and discuss several applications.

Maurizio Paolini

Universitá di Udine, Italy

Approximation of crystalline motion by mean curvature by a nonlinear reaction-diffusion equation

We present some results concerning the so-called crystalline mean curvature flow approximated by a reaction-diffusion equation which formally reads as foll ows:

$$\epsilon^2 \partial_t u - \epsilon^2 \mathrm{div} T(\nabla u) + f(u) = \epsilon g(x,t)$$

where T is a maximal monotone graph, homogeneous of degree 1, describing the underlying crystalline anisotropy; f is the derivative of a bistable potential, and g is a given forcing term. A few numerical simulations suggest good approximation properties similar to those obtained for the classical isotropic mean curvature flow.

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Michael J. Shelley

New York University, USA

INTERFACE INSTABILITY AND DYNAMICS IN NON-NEWTONIAN HELE-SHAW FLOW

The flow of shear-thinning liquids in thin gaps is important to such applications as injection molding (polymeric liquids), and display device design (liquid crystals). From the point of view of pattern formation, the displacement of a shear-thinning liquid by gas in a Hele-Shaw cell can give interfacial patterns – with the suppression of tip-splitting, and appearance of side-branching – that are very reminiscent of anisotropic solidification systems, though no explicit anisotropy is present. To study such problems, we have developed the natural generalization of Darcy's law for non-Newtonian liquids, beginning with fluid mechanical models such as the generalized Navier-Stokes equations, and an Oldroyd fluid model, both of which allow for shear-thinning, and a non-monotonic stress/strain relation. Unlike the Newtonian Darcy's law, the viscosity is now dependent upon the squared pressure gradient, and leads to a nonlinear elliptic boundary value problem for the fluid pressure. We are studying, both analytically and numerically, the modification of the Saffman-Taylor instability of an expanding gas bubble in a shear-thinning liquid. We have shown that shear-thinning leads to enhanced length scale selection. We are also developed numerical methods for simulating the dynamics of the gas/liquid interface, and our simulations show the nonlinear development of patterns composed of stabilized fingers that do not tip-split.

This is joint work with **Ljubinko Kondic** (Courant) and **Peter Palffy-Muhoray** (Liquid Crystal Institute).

Alfred Schmidt

Univ. Freiburg, Germany

A POSTERIORI ERROR ESTIMATION AND ADAPTIVE METHODS FOR STEFAN PROBLEM

We present an adaptive finite element method with a posteriori error estimation for degenerate parabolic problems like the two-phase Stefan problem

$$\partial_t u + \Delta\beta(u) = f.$$

The a posteriori error estimates are based on duality techniques, involving degenerate parabolic problems in nondivergence form. They give computable bounds for the enthalpy error in $L^\infty(0,T;H^{-1}(\Omega))$ and for the temperature error in $L^2(0,T;L^2(\Omega))$. The adaptive method tries to generate quasi-optimal meshes by equidistribution of the local error estimates in space and time.

Numerical results in two and three space dimensions will be presented.

(joint work with **R.H. Nochetto** and **C. Verdi**)

Pierre Gremaud

North Carolina State University, USA

NEW RESULTS IN NUMERICAL CONSERVATION LAWS

In this talk, new results related to the numerical approximation of scalar conservation laws will be presented. In particular, the role played not only by the size of the grid, but also its structure, i.e., uniform vs. nonuniform, Cartesian vs. nonCartesian, will be carefully analysed. The underlying question of the proper notion of consistency will also be discussed. Various new results for specific numerical methods will be presented.

The approach has, among other things, led to an explanation of the supraconvergence phenomenon, and to optimal error estimates in the truly multidimensional case. To the authors' knowledge, both types of results are unique in the framework of conservation laws.

Joint work with **B. Cockburn** and **X. Yang**.

FOCUS SESSION

CAPILLARY FLUID DYNAMICS OF FILM COATING

ORGANIZED BY

Hans Raszillier

Universerstät Erlangen-Nürnberg, Germany

LIST OF SPEAKERS

P.H. Gaskell, The University of Leeds, UK
J. Eggers, Universität Gesamthochschule Essen, Germany
K.D. Danov, University of Sofia, Bulgaria

P.H. Gaskell

The University of Leeds, UK

Stability analysis for reverse roll coating

Two mathematical models are presented for meniscus reverse roll coating in which there is steady flow of a Newtonian fluid in the narrow gap, or nip, between two co-rotating rolls in the absence of body forces.

A model based on lubrication theory and incorporating visco-capillary boundary conditions gives predictions for the pressure field, the meniscus locations and flow rate as a function of roll speed ratio, capillary number and a geometry parameter. Solutions are found to be highly dependent on roll speed ratio and degree of inlet-starvation.

A second model incorporates the nonlinear free-surface boundary conditions and the presence of a dynamic wetting line, adjacent to the web on the upper roll. The latter requires the imposition of an apparent contact angle and a slip length so that numerical solutions for the velocity and pressure fields over the entire domain can be obtained using a finite element formulation of the governing equations of motion. Predictions via both models, for film thicknesses attached to the web and lower roll as functions of roll speed ratio, are in close agreement with each other and also with corresponding experimental data.

Finite element solutions reveal that the flow domain (fluid bead) consists of large recirculations and fluid transfer-jets, and that the flow structure is continuously transformed as the roll speed ratio and inlet flux are varied. For a given capillary number and geometry (roll radius to gap height) two critical flow rates can be identified in the starved/ ultra-starved regimes, each dependent on roll speed ratio, and corresponding to the appearance/ disappearance of: (i) a secondary transfer-jet; (ii) a saddle point at the nip with a downstream sub-eddy. Predicted flow structures and critical flow rates are found to be in qualitative agreement with experimental observation.

Finally, it is shown that, for a given inlet flux, there is a maximum roll speed ratio such that only below this value can two-dimensional steady-state solutions be obtained via the finite element method. It is suggested that this is related to the bead-break instability which arises after the wetting line has passed through the nip. The relevant information is display on an associated control-space diagram.

J. Eggers

Universität Gesamthochschule Essen, Germany

Physics of drop formation

One-dimensional models have proved to be very useful to study the breakup of liquid jets. They can be derived from the Navier-Stokes equation either by expanding the velocity field in the radial direction or by averaging over thin slices of the fluid. The singularity of the equations of motion which describes the radius of the fluid jet going to zero locally is characterized by a balance of surface tension, viscous, and inertial forces. The singular motion is independent of fluid parameters and of initial conditions. Away from asymptotics, one dimensional models are able to describe an even greater variety of scaling behavior. Finally, the mechanisms of main drop and satellite formation are studied in a simulation of a stationary, decaying jet.

K.D. Danov

University of Sofia, Bulgaria

SURFACTANT DYNAMICS IN THIN FILMS

Surfactants play an important role in many technological processes. They adsorb at the interfaces and reduce the interfacial mobility and the surface elasticity due to the change of the interfacial tension. A hydrodynamic model of the multi-component mixture of surfactant including the description of the bulk- and inter-phases is discussed. New constitutive equations for the surface diffusion and heat fluxes, shear and dilatational viscosities are obtained using the Onsager theory for the surface-excess dissipation function. For that purpose the non-equilibrium thermodynamics of a mixture of surfactants in the presence of chemical reactions between them is developed. These relations close the system of equations describing the physical behavior of the moving interface.

The mathematical difficulties arising from the special type of the boundary conditions are illustrated. The possibility of formation of tangential heat, diffusion, and viscous boundary layers on interfaces close to the three phase contact lines are demonstrated. Non-solved mathematical and numerical problems with special technical and biological applications are presented.

The physical approach for integral balance of mass and momentum in surface boundary layers is used to simplify the system of equations. The applicability of this approach is illustrated on the computation of the meniscus profile in coating system. The influence of surface elasticity, viscosity, dynamic and static contact angles, slide velocity, and pressure vacuum on the position of the free three phase contact line is investigated.

FOCUS SESSION

MATHEMATICAL DEVELOPMENT OF FREE BOUNDARY PROBLEMS

ORGANIZED BY

Sandro Salsa
Politecnico di Milano, Italy

LIST OF SPEAKERS

Bjorn Gustafsson, Royal Institute of Technology, Sweden
Stefan Luckhaus, Universität Bonn, Germany
Augosto Visintin, Universitá degli Studi di Trento, Italy

Bjorn Gustafsson

Royal Institute of Technology, Sweden

AN EXPONENTIAL TRNSFORM AND REGULARITY OF FREE BOUNDARIES IN TWO DIMENSIONS

I will discuss a new tool for proving regularity of the free boundary for obstacle type problems with analytic data in two dimensions. Although this is perhaps the simplest nontrivial case of a regularity problem for free boundaries, a complete solution was not given until 1991, by M. Sakai. For the corresponding problem in higher dimension the regularity problem is yet not completely solved, as far as I know. The best known results are due to L. Caffarelli.

The new tool in question is an exponential transform, which is a function of two complex variables originally created by R. W. Carey and J. D. Pincus in an operator theoretic context. Precisely it is

$$E(z,w) = exp[-\frac{1}{\pi}\int_{\Omega}\frac{dA(\zeta)}{(\zeta-z)(\bar{\zeta}-\overline{w})}].$$

Outside the open set Ω, $E(z,w)$ is anlytic in z antianalytic in w and the main result states that if Ω (locally) is the noncoincidence set for the obstacle problem, or if more generally the Cauchy transform of Ω has an analytic extension from outside across $\partial\Omega$, then $E(z,w)$ has an analytic-antianalytic extension, call it $F(z,w)$, across $\partial\Omega$. From this one gets a canonical description of the free boundary $\partial\Omega$ as being the zero set of the real analytic function $F(z,z)$.

One naturally wonders about possibility of generalizing the above ideas to higher dimensions. It does not seem immediate how to proceed, but it certainly is an interesting task for future research.

The connection between the exponential transform and free boundary problems was discovered rather recently by Mihai Putinar, and all results I will present are obtained in collaboration with him.

Stefan Luckhaus

Universität Bonn, Germany

STEFAN PROBLEMS WITH THE GIBBS THOMSON LAW ARE GRADIENT FLOWS FOR THE ENTROPY

Stefan problems in this context are heat and mass diffusion systems with two or more phases.the free boundary is the phase interface. the gibbs thomson law is the additional interface condition,apart from the continuity equations.it says that either the mean curvature of the interface or the difference of mean curvature and velocity is proportional to the jump in free energy across the interface divided by the absolute temperature.this free boundary system can formally be regarded as a gradient flow for the total entropy.this leads to natural approximation schemes,and allows in addition to give a stochastic analogon,a jump process on a finite lattice.

Augosto Visintin

Universitá degli Studi di Trento, Italy

Nucleation and Growth

Surface tension effects occur in several physical phenomena; in particular, they are responsible for the high *undercooling* required for solid *nucleation* in a purely liquid system. (For several metals this undercooling may even reach the order of hundreds of degrees, see e.g. [1,2].)

In a solid-liquid system these effects can be represented by the classical *Gibbs-Thomson law,* which prescribes the relative temperature θ to be proportional to the mean curvature κ of the interface S between phases:

$$\theta = -c\kappa \qquad \text{on } S, \tag{1}$$

see e.g. [3]. Due to the smallness of the coefficient c, this requires the use of a *mesoscopic* length scale (typically of the order of 10^{-5} cm), see e.g. [5].

Evolution is then represented by the law of *mean curvature flow* with a forcing term:

$$av = \theta + c\kappa \qquad \text{on } S \tag{2}$$

where v is the normal velocity of the interface, a is a constant > 0.

Here we want to account not only for smooth surface motion but also for nucleation and other discontinuities in phase evolution. To that aim we replace (2) by an equation of the form

$$\alpha(v) = \theta + c\kappa \qquad \text{on } S \tag{2}$$

where α is bounded, monotone, continuous function $R \to R$ (with $\alpha'(0) = 0$), see [4].

A mesoscopic model of phase transition in two-phase systems is then obtained by coupling the latter equation with the energy balance:

$$\frac{\partial}{\partial t}\left(C_V\theta + \frac{L}{2}\chi\right) - k\Delta\theta = f \qquad \text{in the sense of distributions,} \tag{3}$$

where we set $\chi := -1$ in the solid, $\chi := 1$ in the liquid; C_V is the specific heat and k the heat conductivity.

The resulting problem is *nonconvex,* because of the constraint $\chi = \pm 1$. This is at variance with the weak (macroscopic) formulation of the Stefan problem, see e.g. [6], in which (3) is coupled with the condition

$$\chi = -1 \quad \text{where } \theta < 0, \quad -1 \le \chi \le 1 \quad \text{where } \theta = 0, \quad \chi = 1 \quad \text{where } \theta > 0. \tag{4}$$

Values of χ intermediate between -1 and 1 correspond to a mixture of liquid and solid, a so-called *mushy region;* this is excluded in the previous (mesoscopic) model.

Existence of a solution has been proved for the system consisting of equations (2) and (3) coupled with appropriate initial and boundary conditions, see [7]. In general the uniqueness of the solution is excluded, because of the nonconvexity.

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FOCUS SESSION

FREE BOUNDARY IN FINANCIAL MATHEMATICS

ORGANIZED BY

Thaleia Zariphopoulou
University of Wiscosin, Madison, USA

LIST OF SPEAKERS

Mete Soner,Carnegie Mellon University, USA
Agnes Tourin, Université Paris-Dauphine, France
Ruediger Frey, ETH Zürich, Switzerland

Mete Soner

Carnegie Mellon University, USA

OPTION PRICING WITH CONSTRAINS

I will discuss the problem of determining the minimum cost of super-replicating a claim when there are convex constraints on the portfolio weights. The resulting minimal dominating price is the solution of a quasi-variational inequality. In some interesting examples, such as the European Call option with short selling constraint or the up and out Call option, I will show that the minimal price is equal to the price of a related claim without constraints. This is joint work with **M. Broadie** and **J. Cvitanic** of Columbia University.

Agnes Tourin

Université Paris-Dauphine, France

NUMERICAL APPROXIMATION TO SOME VARIATIONAL INEQUALITIES ARISING IN MATHEMATICAL FINANCE

We consider here several portfolio selection models leading to Variational Inequalities and propose second-order numerical approximations to these Variational Inequalities in order to compute the free boundaries between the transaction regions. The convergence of the schemes based on recent ideas developed by Lions and Souganidis in the framework of first-order Hamilton-Jacobi equations is still an open problem.

This work has been done in collaboration with **T. Zariphopoulou**.

Ruediger Frey

ETH Zürich, Switzerland

OPTIMAL STOPPING: THE FREE BOUNDARY APPROACH AND APPLICATIONS TO DERIVATIVE ASSET ANALYSIS

Optimal stopping problems are of great importance in derivative asset analysis, as they are closely related to the pricing of American options. We therefore take the problem of pricing the American put option as starting point for our talk. We show that the associated optimal stopping problem can be formulated as a free boundary problem for the value function of the American put option. This relationship between free boundary problems and optimal stopping problems is of a general nature. The free boundary formulation will help us to determine the Riesz decomposition of the value process of the option and hence the hedge portfolio for the American put. Finally we sketch a new application of optimal stopping to the superreplication of derivatives in stochastic volatility models: It can be shown that for a large class of European options the value of a superreplicating hedge portfolio is given by the price of the corresponding American option in a model with constant volatility and hence by the value of an optimal stopping problem.

DISCUSSION SESSION

PHASE FIELD MODELS

ORGANIZED BY

Pierluigi Colli
Universitá di Torino, Italy

and

Gianni Gilardi
Universitá di Pavia, Italy

BRIEF INTRODUCTION

In the last decade phase-field models have received a good deal of attention from people working in continuum mechanics and thermodynamics as well as from applied mathematicians experts in PDE. In particular, it is known that phase field problems find noteworthy applications to different phase transition phenomena including solid-liquid phase transitions and martensitic transformations. Moreover, the asymptotic analysis of related systems leads to some interesting and partly unsolved free boundary problems. The aim of this session is then providing the state of the art and reviewing recent results addressed to these problems.

LIST OF SPEAKERS

PHASE-FIELD MODELS - PART 1
Nobuyuki Kenmochi, University Chiba, Japan
Maurizio Grasselli, Politcnico di Milano, Italy
Cedric Dupaix, Universite de Franche-Comte, France
Olaf Klein, WIAS - Berlin, Germany

PHASE-FIELD MODELS - PART 2
Danielle Hilhorst, Université Paris - Sud, France
Elisabeth Logak, Ecole Normale Superieure, France
Reiner Schätzle, Universität Bonn, Germany
Giuseppe Savaré, Istituto di Analisi Numerica del CNR, Italy

PHASE FIELD MODELS - PART 1

This first part is focused on phase transition and phase separation models, collecting both theoretical and numerical results on some PDE's systems which extend the well-known Caginalp and Penrose-Fife approaches. The following talks will be delivered.

Nobuyuki Kenmochi

University Chiba, Japan

PHASE SEPARATION MODEL WITH CONSTRAINT

In his contribution, N. Kenmochi discusses phase separation models which are derived from a class of free energy functionals of Ginzburg–Landau type. These models give rise to a system of two parabolic equations satified by the (absolute) temperature and the conserved order parameter. The first equation comes from the energy balance and is a nonlinear second order parabolic PDE including singularity and at the same time degeneracy, while the second is a nonlinear fourth order parabolic PDE including a maximal monotone graph in the expression of the chemichal potential difference. It will be shown that the main part of this phase separation model can be expressed as the subdifferential of a lower semi-continuous convex function defined on an adequate product space, so that the related problem can be handled within the abstract theory of evolution equations in Hilbert spaces.

Maurizio Grasselli

Politcnico di Milano, Italy

ASYMPTOTIC ANALYSES OF A PHASE FIELD MODEL WITH MEMORY

M. Grasselli deals with a (standard) phase field model in which the heat flux is given by a constitutive assumption of Gurtin-Pipkin type. The resulting nonlinear system which rules the evolution of the relative temperature and of the non-conserved order parameter consists in a hyperbolic integrodifferential equation coupled with a parabolic variational inequality. Several questions related to the well-posedness of the associated Cauchy-Neumann problem have been treated jointly with P. Colli and G. Gilardi. Some asymptotic analysis results are presented in the talk, in particular: (i) convergence to a hyperbolic phase relaxation problem when the interfacial energy coefficient tends to zero; (ii) convergence to a quasi-stationary phase field model when the time relaxation parameter tends to zero; (iii) convergence to a standard parabolic phase field model when the memory kernel tends to the Dirac mass in a suitable way. In all the cases rigorous convergence theorems are proved. Moreover, in (ii) and (iii), some error estimates are also shown.

Cedric Dupaix

Université de Franche-Comte, France

UPPER-SEMICONTINUITY OF THE ATTRACTORFOR A SINGULARLY PERTURBED PHASE FIELD MODEL

The contribution of C. Dupaix is concerned with boundary value problems for a phase field model with polynomial nonlinearity and for two limiting equations, namely the viscous Cahn–Hilliard equation and the Cahn–Hilliard equation. All three problems possess a maximal attractor. The upper–semicontinuity of attractors is shown and a lower–semicontinuity property is presented for the one–dimensional viscous Cahn–Hilliard equation. These results arise from a collaboration with D. Hilhorst, I. Kostin, and Ph. Laurençot. Morever, in the talk the case of a logarithmic nonlinearity (which is more realistic in some contexts such as solid–solid phase transitions) is considered for the phase field model and the upper–semicontinuity of the global attractor is extended to this framework.

Olaf Klein

WIAS - Berlin, Germany

A PHASE-FIELD SYSTEM WITH SPACE-DEPENDENT RELAXATION COEFFICIENT

O. Klein considers a phase–field system of Penrose–Fife type, with a space–dependent relaxation coefficient. A convergent time–discrete scheme is discussed for the approximation of the corresponding initial–boundary value problem. Existence, uniqueness, and convergence results are derived, even for a system with degenerate relaxation coefficient. In conclusion, some numerical methods and tests are presented.

PHASE FIELD MODELS – PART 2

The second part is mainly devoted to the asymptotic behaviour of PDE's systems with respect to suitable kinetic parameters and to the characterization of the limit problems and of their solutions. Indeed, the concerned titles are as follows.

Danielle Hilhorst

Université Paris - Sud, France

A CHEMOTAXIS–GROWTH MODEL

D. Hilhorst analyses an advection–reaction–diffusion system proposed by Mimura–Tsujikawa and describing chemotaxis with growth. Chemotaxis is a phenomenon observed in biology, in which biological individuals migrate towards higher gradients of some chemical substance – the chemotactic substance – which is produced by themselves. The two unknown functions are the density of biological individuals and the concentration of chemotactic substance. As a small parameter tends to zero, the solution of the chemotaxis problem converges to the solution of a free boundary problem, in which the motion of the interface is partly induced by its mean curvature. The limiting moving boundary problem possesses a unique smooth solution locally in time. Such convergence for the solutions of the chemotaxis–growth problem has been shown in a joint work with A. Bonami, E. Logak and M. Mimura. In addition, it has been pointed out that properties such as the inclusion and the convexity of interfaces are not conserved in time.

Elisabeth Logak

Ecole Normale Superieure, France

MASS CONSERVED ALLEN-CAHN EQUATION AND AREA PRESERVING MEAN CURVATURE FLOW

A mass conserved Allen–Cahn equation is considered by E. Logak. This equation has been proposed by Rubinstein and Sternberg to describe phase separation in binary mixtures and it can be viewed as a limit of the viscous Cahn–Hilliard equation. The existence of a unique smooth solution to the area preserving mean curvature flow is assumed in some time interval. Under this condition, we will see that the solutions of the mass conserved Allen–Cahn equation converge to this limiting motion problem, this result following from a cooperation with X. Chen and D. Hilhorst.

Reiner Schätzle

Universität Bonn, Germany

MEAN CURVATURE FLOW APPROXIMATED BY THE EXTENDED

Schätzle communicates on a joint work with D. Hilhorst and L. Peletier regarding the Fisher–Kolmogorov equation $\partial_t u + \varepsilon^2 \gamma \Delta^2 u - \Delta u + \varepsilon^{-2} F'(u) = 0$. Here, $F(t) := (t^2 - 1)^2/4$ is a double–well potential and, for $\gamma = 0$, the above PDE reduces to the ordinary Allen–Cahn equation. Formal asymptotics show that the limit as $\varepsilon \to 0$ yield the mean curvature flow. But a difficulty arises since it is not clear whether the stationary waves (that is, the solutions of the ODE $\gamma U'''' - U'' + F'(U) = 0$ with appropriate boundary conditions) are unique. There could also be multi–bump solutions. Peletier and Troy have proved that monotone and odd solutions are unique when $0 < \gamma \leq 1/8$. The main result of this talk establishes that the area functional is the Γ–limit of ε times the stationary energy functional associated with the above Fisher–Kolmogorov equation. Also, in one space dimension the limit interfaces move with constant velocity.

Giuseppe Savaré

Istituto di Analisi Numerica del CNR, Italy

COMPACTNESS PROPERTIES FOR THE ASYMPTOTICS OF SOME PHASE FIELD EQUATIONS

The talk of G. Savaré addresses the following typical problem, which occurs in passing to the limit in some phase field models: for two families of space–time dependent functions $\{u_\varepsilon\}$, $\{v_\varepsilon\}$ (representing, e.g., temperature and phase variable) we know that the sum $u_\varepsilon + v_\varepsilon$ converges in some L^p–space and that the time integrals of a suitable space–functionals evaluated on u_ε, v_ε are uniformly bounded with respect to ε. Can we deduce that u_ε and v_ε converge separately? Luckhaus (1991) gave a positive answer to this question in the framework of the two–phase Stefan problem with Gibbs–Thompson law for the melting temperature. Plotnikov and Starovoitov (1993) proposed a possible general solution employing the original idea of Luckhaus and a space compactness assumption. We show how this problem is related to general metric and scaling properties of the functionals and the topologies involved, providing sufficient conditions for its solvability. This approach allows simp le applications to concrete situations not covered by the previous results, like phase–field equations with Neumann boundary conditions on the temperature.

DISCUSSION SESSION

FREE BOUNDARY PROBLEMS IN THE THEORY OF FILTRATION

ORGANIZED BY

S. Antontsev
Universidade da Beira Interior, Portugal

and

D. Hilhorst
Université Paris - Sud, France

BRIEF INTRODUCTION

The study of filtration problems is very important both from the theoretical as from the application points of view. There are natural applications in environmental sciences when modelling pollution contaminants and their transport by groundwater and in petrol engineering. The mathematical models which arise lead to difficult problems for systems of nonlinear partial differential equations and free boundary problems. The scope of this session is to discuss the construction of mathematical models for multiphase filtration. Such topics were also discussed in the previous congresses of this series and are far from being completely understood from the mathematical point of view. Questions such as existence, uniqueness and qualitative properties of the solutions of these free boundary models as well as their numerical computation will be raised. The speakers will be the following.

LIST OF SPEAKERS

S.N. Antontsev, Universidade da Beira Interior, Portugal
M. Chipot, Universität Zürich - Irchel, Switzerland
M. Guedda, Université de Picardie Jules Verne, France
J. Kacur, MFF Comenius University, Slovakia
R. Kersner, Hungarian Academy of Sciences, Hungary
S. Shmarev, Universidad de Oviedo, Spain

S.N. Antontsev

Universidade da Beira Interior, Portugal

SMALL PARAMETERS IN THE PROBLEMS OF FILTRATION OF TWO IMMISCIBLE FLUIDS

This talk, which describes joint work with **G. Gagneux**, is devoted to the investigation of qualitative properties of weak solutions of the system

$$\frac{\partial s}{\partial t} = (\varepsilon_1 a(s)\nabla s) + b'(s)\nabla p \nabla s, \quad (a(s) \geq 0, \quad s \in [0,1])$$

$$\varepsilon_2 \frac{\partial p}{\partial t} = (k(s)\nabla p)\,, \quad (\varepsilon_i > 0, \varepsilon_i \to 0)$$

where $\varepsilon_i > 0$ are small parameters. The system models the filtration of two immiscible weakly compressible fluids [1], [2], [3].

The following qualitative properties of solutions are studied [3]:
support properties of weak solutions ($\varepsilon_i > 0$);
passages to the limit in $\varepsilon_i \to 0$.

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M. Chipot

Universität Zürich - Irchel, Switzerland

ON THE DAM PROBLEM WITH LEAKY BOUNDARY CONDITIONS

We would like to present new results -in particular of uniqueness- for the dam problem with leaky boundary conditions. A so called leaky boundary condition is a condition of non linear flux on the bottom of the different reservoirs considered.

M. Guedda

Université de Picardie Jules Verne, France

Blow up and regularity of interfaces for an inhomogeneous filtration equation

The evolution of interfaces between fresh and salt groundwater in aquifers in coastal areas can be modelled by the equation

$$\rho(x)u_t = (u(1-u)u_x)_x$$

where u represents the height of salt water. We first give a sufficient condition on the function ρ for the blow up of the support of solutions of namely $\{(x,t), \quad 0 < u(x,t) < 1\}$. Similar results have been obtained by Kamin, Kersner and Galaktionov about the support of solutions of the equation

$$\rho(x)u_t = (u^m)_{xx}$$

We then present regularity results for interfaces of solutions of This is joint work with **D. Hilhorst**, **M. A. Peletier** and **S. I. Shmarev**.

J. Kacur

MFF Comenius University, Slovakia

Solution of degenerate convection-diffusion problems by the method of characteristics

A new approximation scheme is proposed to solve a nonlinear degenerate convection-diffusion problems. The method is based on relaxation schemes developed by the author in previous papers and on the method of characteristics initiated by O. Pironneau, J. Douglas and T.F. Russel. The mathematical model includes a nonlinear degenerate parabolic term and also a nonlinear convective term. In the elliptic term the diffusion coefficient depends on the unknown function. The convergence of the method is proved and numerical experiments supporting the theoretical results are presented. These problems model the transport of reactive contaminant under Freundlich sorption isotherm.

R. Kersner

Hungarian Academy of Sciences, Hungary

Interface blow-up in inhomogeneous media

The model problem

$$\rho(x)u_t = (u^m)_{xx} - c_0 u^n,$$

$$u(x,0) = u_0(x)$$

will be considered, where $m, n > 1$, c_0 nonnegative and $\rho(x) \sim |x|^{-l}$ for large $|x|$.

The initial datum $u_0(x)$ is nonnegative and has compact support.

We address the following questions: when does the solution remain compactly supported for all time and when it becomes everywhere positive after some finite time.

Two-sided estimates for interfaces will also be given. Joint work with **A.Tesei** (Rome).

S. Shmarev

Universidad de Oviedo, Spain

On the regularity of interfaces in solutions of degenerate parabolic equations. Technique of intersection comparison

We propose a new approach for the study of questions of regularity of solutions and interfaces for a class of degenerate parabolic equations with one space variable. The method is presented in the framework of the study of equations of the form

$$u_t = (u^m)_{xx} - u^p \tag{1}$$

in the range of parameters $m > 1$, $0 < p < 1$, subject to the condition $m + p - 2 \geq 0$. It is based on the idea that the local analysis of solutions of equation (1) near an interface can be done by means of *intersection comparison* with the family of *travelling wave solutions* of the equation. The comparison technique called Intersection Comparison or Lap Number roughly says that the number of sign changes between two solutions of equations like (1) is nonincreasing in time. We are able to show in this way that the behaviour of the solutions near the interface, and in particular its local velocity, are determined in first approximation by a travelling wave profile.

• We prove that for continuous, compactly supported and bell-shaped initial data the solutions of the Cauchy problem for equation (1) have Lipschitz continuous interfaces in any compact time interval $0 < t_1 \leq t \leq t_2 < T_e$. Here $T_e < \infty$ is the instant when the solution extincts.

• We also obtain the formulae which express the interface velocity in terms of the solution profile, so-called *interface equations* , for both expanding and receding waves. It is shown that at each instant $t \in (0, T_e)$ the function is right and left differentiable.

The method can be applied in principle to quite general diffusion-absorption equations of the form with monotone nondecreasing functions and other similar equations with convective or reactive terms, or with other types of nonlinear diffusion.

The talk contains results of joint work with **V.A.Galaktionov** and **J.L.Vazquez**.

DISCUSSION SESSION
MATHEMATICAL FREE BOUNDARY PROBLEMS IN GLACIOLOGY

ORGANIZED BY

Kolumban Hutter
Technische Hochschule - Darmstadt, Germany

Luisa Santos
Universidade do Minho, Portugal

and

Brian Straughan
Glasgow University, UK

BRIEF INTRODUCTION

We propose to present a discussion in various free boundary problems arising in the geophysical context of glaciology involving permafrost and its melt-phase, ice sheet and ice shelf dynamics, and subglacial till behaviour. Emphasis is placed on demonstrating the occurrence of free moving boundaries, internal and interface behaviour, phase changes and, partly, mushy zone behaviour. We outline open mathematical problems, review the pertinent physical literature and sketch possible problems worthy of further mathematical investigation.

LIST OF SPEAKERS

J.I. Diaz, Univ. Complutense de Madrid, Spain
K.H. Hutter, Technische Hochschule - Darmstadt, Germany
F. dell'Isola, Universita di Roma "La Sapienza", Italy
J.M. Urbano, Universidade de Lisboa, Portugal

J.I. Diaz

Universidad Complutense de Madrid, Spain

ON A PARABOLIC-HYPERBOLIC COUPLED SYSTEM ARISING IN GLACIOLOGY

The mechanism whereby large ice sheets can surge periodically, (Heinrich events, Hudson bay phenomenos, Canada) was recently studied by Fowler and Johnson (1995). They proposed a two-dimensional ice sheet simplified model that includes basal ice sliding dependent on the basal water pressure, which essential consists in the following system:

$$h_t - \left[(\delta+Q)^S h^{R+1}|h_x|^{R-1}h_x\right]_x = a \quad in\ Q_T \tag{1}$$

$$\begin{aligned} &Q_x - (\delta+Q)^S\left[h^{R+1}|h_x|^{R+1} + \beta h^R|h_x|^{R-1}h_x\xi^{-1/2}\right] - \gamma + \lambda h^{-1} \geq 0,\ Q>0 \quad in\ Q_T \\ &\left(Q_x - (\delta+Q)^S\left[h^{R+1}|h_x|^{R+1} + \beta h^R|h_x|^{R-1}h_x\xi^{-1/2}\right] - \gamma + \lambda h^{-1}\right)Q = 0 \qquad in\ Q_T \end{aligned} \tag{2}$$

$$\xi_x = (\delta+Q)^S h^R|h_x|^R \qquad in\ Q_T \tag{3}$$

the unknown variables are the ice depth h, the accumulated ice velocity ξ, and the basal water flow Q. Prescribing suitable boundary conditions for h, ξ, Q and an initial condition for h Fowler and Schiavi (1997) solved numerically (1), (2) and (3) using a fully implicit backwards finite difference scheme for h and an improved Euler method for Q and ξ.

The main goal of this work is to present some results on the existence of solutions for the implicit backwards scheme system and some numerical experiences (Fowler and Schiavi, 1997) for some special data illustrating the surging mechanism.

Joint work with **E. Schiavi.**

K.H. Hutter

Technische Hochschule - Darmstadt, Germany

THE MATHEMATICAL FOUNDATION OF ICE SHEET AND ICE SHELF DYNAMICS

A physicist's view of the pressing problems: We briefly describe the mathematical formulation of the boundary value problems that arise in global ice sheet dynamics and describe the simplest models of the ice sheet/shelf responses to climate variations. We show in particular for cold ice and for polythermal ice which free boundary value problems the geophysicist would like to solve and what complications would have to be overcome, to asure that the problems are well posed. We pose questions and ask mathematicians for answers.

F. dell'Isola

Universita di Roma "La Sapienza", Italy

A FREE BOUNDARY VALUE PROBLEM FOR THE TILL BELOW LARGE ICE SHEETS: WHY IS THE MATHEMATICAL PROBLEM PHYSICALLY RELEVANT?

J.M. Urbano

Universidade de Lisboa, Portugal

ON THE MATHEMATICAL ANALYSIS OF A VALLEY GLACIER MODEL

We study a free boundary problem arising from theoretical glaciology:

$$\begin{aligned} \Delta_p u_\xi &= -1 \quad \text{in } \Omega_\xi \\ u_\xi &= 0 \quad \text{on } \Gamma_H^\xi \\ \frac{\partial u_\xi}{\partial z} &= 0 \quad \text{on } \Gamma_L^\xi \end{aligned}$$

From the mathematical point of view it consists of a mixed boundary value problem for the p-Laplacian in a domain Ω_ξ that is not known *a priori* and is to be determined so that a prescribed mass flux condition is satisfied, namely $\int_{\Omega_\xi} u_\xi = Q$. Our result establishes the existence of a unique weak solution to the problem by means of the careful analysis of the range of an appropriate real function.

Joint work with **J. F. Rodrigues**.

DISCUSSION SESSION

DYNAMICS OF SURFACE GROWTH AND PHASE TRANSITION KINETICS

ORGANIZED BY

Ivan Götz

Technische Universität München, Germany

and

Boris Zaltzman

Ben-Gurion University of Negev, Israel

BRIEF INTRODUCTION

This session is to address the interface dynamics in phase change and crystal growth on the following two aspects. The first are the kinetic effects, either in their proper thermodynamic context or as a tool of regularization of ill-posed problems (Ch. Charach, P. Fife and I. Götz). The other aspect of phase interface dynamics to be addressed is that of grain at the surface of a polycrystalline material (A. Vilenkin and B. Zaltzman). This dynamics is modelled by evolutionary fourth order equations in a possible combination with additional "kinetic" equations, which have so far attracted relatively little attention by the FBP community.

LIST OF SPEAKERS

A. Vilenkin, Hebrew University of Jerusalem, Israel
B. Zaltzman, Ben-Gurion University of Negev, Israel
I. Götz, Technische Universität München, Germany
A. Novick-Cohen, Technion - IIT, Israel
P. Rybka, Technische Universität München, Germany
W. Merz, Technische Universität München, Germany

Arkady Vilenkin

Hebrew University of Jerusalem, Israel

EFFECT OF SURFACE DYNAMICS ON PHASE TRANSFORMATION AND MASS TRANSFER

Three different problems, combining surface dynamics with interface motion or external mass flux through the interface are discussed. The first describes the motion of the phase boundary in two-dimensional system with free surfaces corresponding to grooves around a triple junction. The related free boundary problem consists of three partial derivative equations coupled through boundary conditions. In the long time limit the problem has depending on input data either two stable traveling wave solutions, or a life time of the system is limited. The second problem concerns the motion of grain boundary in a two-dimensional bicrystal with anisotropic free surfaces. Dufference in surface energy of neighboring grains yields grain boundary motion, resulting in decreasing of systems energy. Mathematical structure of this problem is similar to that of the previous problem. In the long time limit is studied. In this limit, depending on the input data, the solution is either a stable traveling wave or does not exist at all. A peculiar feature of this traveling wave solution is a related "negative differential resistance" of the grain boundary velocity dependence on the driving force. The third problem describes the motion of the free surface of a two dimensional bicrystal placed between two rigid walls. The respective mathematical formulation consists of a fourth order partial differential equation for surface dynamics with two free boundaries and a restriction on the maximum value of the surface shape function (the requirement of non-overlap of the two crystals). The problem possess a steady-state solution whenever the control parameter (a specified mass flux along the grain boundary due to an external force) is below some critical value. For larger fluxes the solution oscillates in time, with each oscillation generating a void, which propagates along the grain boundary from the surface into the bicrystal. This change of morphology decreases the effective value of the flux imposed on the free surface.

Boris Zaltzman

Ben-Gurion University of the Negev, Israel

ASYMPTOTIC SOLUTIONS TO THE FREE BOUNDARY PROBLEM ARISING IN THE GRAINS DYNAMICS

The boundary value problem, arising in the modelling of the motion of grain boundary in a two-dimensional bicrystal, has been studied. The studied problem is a free boundary problem for a fourth-order partial differential equation. The well-posedness of this free boundary problem has been studied and the short time asymptotic behavior of the solution analysed.

I. Götz

Technical University of Munich, Germany

"UNDERCOOLING" IN TYPE I SUPERCONDUCTORS

We consider a one-dimaensional model of phase changes from "normal" to superconducting

(see [1], [2]).

$$
\begin{aligned}
&u_t - u_{xx} = 0, \quad 0 < x < s(t),\\
&v_t - v_{xx} = 0, \quad 0 < x < s(t),\\
&u_x = -s'u \quad \text{on} \quad x = s(t),\\
&v_x = -s'v \quad \text{on} \quad x = s(t),\\
&u^2 + v^2 = H_c^2 \quad \text{on } x = s(t),\\
&u_x = v_x = 0, \quad \text{on} \quad x = 0;\\
&u(x,0) = u_0(x), \; v(x,0) = v_0(x),
\end{aligned}
$$

here $\vec{H} = (u, v)$ is the magnetic field and the graph of the function s separates the superconducting region $\{x > s(t)\}$ from the normal one $\{0 < x < s(t)\}$. A. Friedman and B. Hu [2] studied this problem under the assumption, that the "normal" region is continuously increasing in time, i.e. the superconducting region shrinks. We are interested in the opposite case, when the superconducting region is increasing in time. This problem is closely related to the usual one-phase scalar Stefan problem with undercooling (consider $v \equiv 0$). That problem has been studied in the paper [3]. We have proved, that under some critical undercooling a blow-up can develope and the free boundary becomes discontinuous. The question was: whether the similar phenomena are usual for the whole problem (1)-(7). Our answer is : NO. Such blow-ups are rather an exeption for our vector valued problem (1)-(7). More exactly: if the function v_0 is more than some positive constant, and some smoothness assumptions are satisfied, then the function s is Hölder continuous and piece-wise infinite times differentiable. In order to get a solution we take a kinetic regularization:

$$u^2 + v^2 = H_c^2 + \epsilon s' \quad \text{on } x = s(t),$$

instead of (5). Then we obtain some independent on ϵ estimates for the functions u, v, $w = u^2 + v^2$, $\phi = arctan(v/u)$ and tend ϵ to zero.

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Chaim Charach (Ben-Gurion Univ. of the Negev) and Paul C. Fife (Univ. of Utah) "Thermodynamically consistent phase-field models of solidification processes"

Amy Novick-Cohen

Technion - IIT, Israel

Order-Disorder, Phase Separation, and Wetting

A system of Cahn-Hilliard/Allen-Cahn equations are considered which model simultaneous order/disorder and phase separation. It is demonstrated that this system exhibits the possibility of triple junction motion in which the interface between the two ordered phases is completely wetted by the disordered phase.

Piotr Rybka

Technische Universität München, Germany

ASYMPTOTICS FOR AN EQUATION RELATED TO MARTENSITIC PHASE TRANSITIONS

We study asymptotic behavior of

$$0 = \mathrm{div}\sigma(\nabla u) + \Delta u_t - \delta^2 \Delta^2 u, \tag{1}$$

where $u : \Omega \subset IR^2 \to IR$, $\sigma(\xi) = W(\xi)$ and W has several local minima. This equation is the quasi-steady approximation of the viscoelasticity with capillarity

$$\rho u_{tt} = \mathrm{div}\sigma(\nabla u) + \Delta u_t - \delta^2 \Delta^2 u, \tag{2}$$

Setting $\rho = 0$ corresponds to neglecting the inertial effects and it is justifiable for slow processes.

Equation (1) arises in the studies of van der Waals fluids and phase transitions in solids. Its applicability to models of moving phase boundaries has been justified by Slemrod and more recently by Abeyaratne – Knowles in the context of phase transitions in solids. We adopt special boundary conditions, such that the influence of the boundary on the overall behavior is little. We set $\Omega = [0, \omega] \times [0, L]$ and

$$u_{x_2} = \Delta u_{x_2} = 0 \quad \text{at } x_2 = 0, L, \qquad u \text{ is anti-periodic in } x_1. \tag{3}$$

We define the energy functional to be

$$E(u) = \int_\Omega [W(\nabla u(x)) + \frac{\delta^2}{2} |\Delta u(x)|^2] \, dx,$$

where $W(F_1, F_2) = \phi(F_1) + \frac{1}{2} F_2^2$, and $\phi \in C^\infty(IR)$ is such that, $\phi(t) \geq 0$, $\phi(t) = 0$ iff $|t| = 1$, and $\phi(t) = \phi(-t)$.

Our main result is

Theorem. *Let us suppose that p is large, W satisfies the conditions above and in addition ϕ is real analytic with polynomial growth. If $u_0 \in W^{3,p}(\Omega)$ is anti-periodic satisfying the first of boundary conditions (3), then there exists a unique strong solution of (1) which satisfies (3). Moreover, if in addition its energy $E(u_0)$ is sufficiently small, then $u(t)$ converges in $W^{4,p}(\Omega)$ to a minimizer of E.*

Let us point out that because of the boundary condition we have a circle of minimizers. The proof of this Theorem starts with a study of critical points of E. We show that global minimizers do not depend on x_2. This circle of minimizers is separated from other critical points of E in the topology of $W^{2,p}(\Omega)$, $p \geq 2$. It turns out that $\delta^2 E/\delta u^2$ has a two-dimensional kernel at the minimizers. Thus, standard tools of the theory of dynamical systems are not applicable here. So, we resort to methods based on analyticity of W and relying on a Łojasiewicz' type inequality.

Joint work with **Karl-Heinz Hoffmann**.

Wilhelm Merz

Technische Universität München, Germany

CONSTRUCTION OF AN ASYMPTOTIC MODEL FOR THE OXIDATION PROCESS OF SILICON

The simulation of thermal oxidation, an important process step in integrated circuit production, requires the computation of a change of layer structures with time. At a process temperature which typically lies between 700–1200^{0}C, the silicon wafers are exposed for 30–500 minutes to a gas atmosphere containing oxygen (O_2). Oxygen enters the silicon-dioxide layer (SiO_2) from outside, diffuses through it towards the silicon (Si) and reacts there to produce new silicon-dioxide according to $\mathrm{Si} + \mathrm{O}_2 \to \mathrm{SiO}_2$.

As silicon is consumed, the boundary layer between silicon and silicon-dioxide moves further into the silicon. The freshly created oxide requires more than twice the volume of the consumed silicon, for which reason the surface of the oxide layer grows outwards. The reaction rate and the volume expansion are usually non-uniform along the reaction interface and this results in non-homogeneous deformations of the whole layer system.

There exists a variety of mathematical models describing this process. We consider a model, where the two phases, silicon and silicon-dioxide, are separated by a moving mixed zone of thickness $O(\varepsilon)$, ε being a small parameter. We use the techniques of formal asymptotic analysis to derive a model with a sharp reaction front between the two phases. The resulting free boundary problem agrees with other existing oxidation models on the basis of a sharp interface. In addition we give a proof for the existence, uniqueness and qualitative behaviour of the solution to the system of ODEs obtained to leading order of the interfacial expansion.

CONTRIBUTED TALKS

Robert Almgren

University of Chicago, USA

Second-order phase field asymptotics for unequal conductivities

The phase field method constructs a pair of stiff reaction-diffusion PDEs whose solutions have a thin internal layer. The PDE system is constructed so that as a parameter in the equations tends to zero, the layer approaches a sharp interface, satisfying the desired boundary conditions at leading order, and coupling to the diffusion equation in the bulk. In numerical computations, it is very difficult to make the interface thin enough to achieve quantitative agreement with the sharp-interface model. Recently, Karma and Rappel have obtained accurate results by taking account of temperature variations within the interfacial layer and by slightly modifying the kinetic coefficient in the PDE. We show how their construction is in effect a *second-order* asymptotic analysis, and we extend it to the case when the thermal conductivities are different in solid and liquid, as for the very important problem of alloy solidification. We hope that second-order asymptotic analysis, and identification of the leading-order error terms, will soon be considered to be as fundamental as are their analogs for numerical discretization schemes.

Zhimirg Chen

Academia Sinica, China

Numerical methods for two phase continuous casting problems

The continuous casting problem is a mathematical model describing the solidification of a material being cast continuously with a prescribed velocity. We will propose a finite element scheme which is based on an implicit discretization of the diffusion and an explicit approximation of the convection. For the problems with nonlinear flux boundary conditions we are able to show the converegence of the method. The analysis is based on a new apriori estimate of fractional derivative in time of the discrete temperature. For the problems with linear flux boundary conditions we are able to derive an error estimate of the same convergence order as that analysis is based on a new sharp boundary estimate for the Green operator under the condition that the free boundary does not touch on the inflow boundary.

G. Georgiou

University of Cyprus, Cyprus

Converged solutions of the planar and axisymmetric extrudate swell problems

We solve the planar and axisymmetric Newtonian extrudate swell problems using standard and singular finite elements. We study the convergence of the two methods with mesh refinement, for various values of the Reynolds and capillary numbers. The singular finite elements are superior to the standard finite elements only when coarse or moderately refined meshes are used, the Reynolds number is low and the surface tension is not high. The standard finite elements perform better as the surface tension and/or the Reynolds number are increased, which implies that the effect of the stress singularity at the exit of the die becomes less important.

Joint work with **Andreas G. Boudouvis** (National Technical University of Athens).

Marianne K. Korten

University of Buenos Aires, Argentina

A STRUCTURE THEOREM FOR SOME FREE BOUNDARY PROBLEMS FOR THE HEAT EQUATION

In one space dimension and for a given initial datum $u_I(x) \in C_0^\infty$, (say such that $u_I(x) > 1$ in some interval) the equation $u_t = \Delta(u-1)_+$ can be thought of as describing the energy per unit volume in a Stefan-type problem where the latent heat of the phase change is given by $(1-u_I(x))_+$.

Given a solution in the sense of distributions $0 \leq u \in L^1_{\mathrm{loc}}(\mathbb{R}^n \times (0,T))$ to this equation, we prove that the free bounbary is n-rectifiable, i. e., but a null set the union of images of imbedded C^1 manifolds of dimension n of $\mathbb{R}^n \times (0,T)$. On the free boundary the "classical" Stefan condition is shown to hold pointwise.

We show the same result for the combustion model.

A. Myslinski

System Research Institute, Poland

FICTITIOUS DOMAIN APPROACH FOR SOLVING SHAPE OPTIMIZATION PROBLEMS

The paper deals with the numerical solution of a shape optimization problem of an elastic body in unilateral contact with a rigid foundation. The equilibrium state of this contact problem is described by an elliptic variational inequality of the second order.

The shape optimization problem for the elastic bodies in contact consists in finding, in a contact region, such shape of the boundary of the domain occupied by the body that the normal contact stress is minimized. It is assumed that the volume of the domain occupied by the body is constant. Moreover the function describing the boundary of the domain occupied by the body and its derivative are bounded.

Shape optimization of elastic or hyperelastic contact problems was considered, among others in [r5,r6,r7,r9,r11]. The existence of optimal solutions was investigated in [r9]. The neccessary optimality conditions were formulated in [r11] using the material derivative approach as well as in [r5,r9] using the optimal control theory approach. The convergence of the finite element approximation was investigated in [r6,r9]. Numerical results are reported in [r6,r7,r9]. In these papers the classical approach to numerical solving of optimal shape design problems based on boundary or domain variations methods [r4,r8,r10] was used. In this approach the state problem is solved many times on the domain changing during the computation. The boundary or domain variation methods require calculation of a new triangularization of the optimized domain, updating the stiffness matrix and load vector at each iteration of the numerical algorithm. Since the optimized domain usually has complicated geometrical structure the whole computational process is tedious, time consuming and expensive. To overcome this difficulty, in response to growing number of industrial applications of the optimal shape design problems, fixed domain methods [r8] for solving these problems are being developed. Fixed domain methods are based on using fictitious domain method [r2,r3]. The fictitious domain method for solving the state systems described by partial differential equations consists in transforming the original state system defined in the complicated geometry domain into a new system defined in a given fixed simply geometry domain containing the original domain with the same differential operator [r2,r3,r4]. This method allows to use fairly structured meshes on a simple geometry domain containig the actual one and fast solvers.

Fixed domain methods for solving shape optimal design problems were considered in [r4,r8,r10]. In [r4] the original shape optimal design problem for the Laplace state equation with Dirichlet boundary condition was transformed into equivalent one using a new control variable on the right-hand side of the state equation defined in the fixed domain containing the original one. The

convergence of the finite element approximation is shown as well as a numerical result obtained by using nonsmooth optimization method is presented in [r4]. In [r8] the original shape optimal design problem is replaced by a constrained control problem with either boundary control or distributed control. The state system is defined on a fixed larger domain than the original one. The existence of optimal controls and optimality conditions are investigated in [r8]. The numerical algorithm based on the fixed domain method for shape optimization of flow problems is proposed in [r10].

This paper deals with the numerical solving of a shape optimization problem for linear contact problem using the fictitious domain approach. To author knowledge this approach for contact problems has not been investigated yet and the present paper is continuation of the author paper [r6]. In the paper we shall formulate 2D contact problem in variable domain. The boundary of the domain is variable subject to optimization. The cost functional approximating the normal contact stress is introduced [r6,r7]. The fictitious domain formulation to this optimization problem is given using the boundary control technique [r3,r9] and the equivalence to the classical formulation is shown. The piecewise linear finite element approximation for displacements and constant for Lagrange multipliers is proposed and its convergence is shown. The design sensitivity analysis for the discrete shape optimization problem is performed. The Lagrangian multiplier method for solving the discretized state system is proposed [r7]. The descent gradient method is used for solving the optimization problem. Numerical examples are provided and discussed.

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SINGULAR LIMITS OF SOLUTIONS TO CAHN-HILLIARD EQUATION AND AN EVOLUTION OF MICROSTRUCTURES

We deal with the singular limits, as small parameters $(\tau, \varepsilon) \to 0$, to boundary-valued problem

$$(\lambda\vartheta - \frac{u^2}{2})_t = \vartheta_{xx}, \quad \tau u_t = \varepsilon^2 u_{xx} - u^3 - \vartheta u \quad (0 < x < 1,\, 0 < t < T),$$

$$u(0,t) = u(1,t) = \vartheta(0,t) = \vartheta(1,t) = 0,$$

$$u(x,0) = u_0(x), \quad \vartheta(x,0) = \vartheta_0(x), \quad (0 < x < 1).$$

which may be regarded as linearization of Penrose-Fife model near the critical temperature. It is supposed that $\tau = \varepsilon^\alpha$, $\alpha \in (1,2)$ and initial data satisfy the following conditions:

$$\varepsilon \|u_{0x}\|_{L_2(0,1)} + \|u_0, \vartheta_0\|_{C(0,1)} \leq c, \quad u_0, \vartheta_0 \in C^2(0,12).$$

Under these assumptions the problem has the unique smooth solutions $(\vartheta_\varepsilon, u_\varepsilon)$. The basis idea is that a description of the passage to limit can be down in terms of slow variables: the temperature ϑ_ε and the adiabatic invariant $I_\varepsilon = 2_1(u_\varepsilon^2 + \vartheta_\varepsilon)^2 - \varepsilon^2 u_{\varepsilon,x}^2$. The main result is the following:

Theorem *Let $(u_\varepsilon, \vartheta_\varepsilon) \to (u, \vartheta)$ weakly in $L_2(Q)$. Then:*
a)$(u_\varepsilon, \vartheta_\varepsilon) \to (u, \vartheta)$ strongly in $L_2(Q)$. and the inclusion

$$(I_\varepsilon, \vartheta_\varepsilon) \in \{I, \vartheta : \vartheta < 0, 0 < i < \vartheta^2\} \cup \{I = \vartheta^2\}.$$

holds almost everywhere in $Q = (0,1) \times (0,T)$

b) Whenever continuous even function $g(u)$,

$$w - limit\, g(u_\varepsilon) = \int_{-\infty}^{\infty} K(z, I, \vartheta) g(z) dz$$

where K is some algebraic function.

B. Stoth

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A DEGENERATE PARABOLIC ELLIPTIC SYSTEM AND A FREE BOUNDARY PROBLEM

In a Type–II superconductor the magnetic field penetrates the superconducting body through the formation of vortices. Because the number of vortices is large it seems feasible to model their evolution by an averaged problem, known as the mean field model of superconductivity. In this talk I will discuss a two dimensional reduction, assuming all vortices to be perpendicular to a given direction. Since both the magnetic field H and the averaged vorticity ω are curl free, they may be represented via a scalar magnetic potential q and a scalar stream function ψ, respectively. I will present existence and uniqueness results of solutions (ψ, q) of the resulting degenerate elliptic parabolic system

$$\psi_t - |\nabla\psi|(\sigma\nabla \cdot (\tfrac{\nabla\psi}{|\nabla\psi|}) + q - \psi) = 0 \quad \text{in } \Omega\times]0,T[,$$
$$-\Delta q + \chi_\Omega q = \chi_\Omega \psi \quad \text{in } I\!R^2\times]0,T[,$$

by means of viscosity and weak solutions. In addition I will relate (ψ, q) to solutions (ω, H) of the mean field equations. Finally I will discuss the regularity properties of the corresponding stationary free boundary problem and present special solutions to it.

This is joint work with **C. M. Elliott** and **R. Schätzle**.

D. Tarzia

Univ. Austral, Argentina

NUMERICAL ANALYSIS OF THE STEADY-STATE TWO-PHASE STEFAN-SIGNORINI PROBLEM

We consider a steady-state heat conduction problem in a multidimensional bounded domain with regular boundary. We assume, without loss of generality, that the melting temperature is zero degree centigrade. We consider a source term g in the domain, a convective condition on a part B1 of the boundary, a positive heat flux q on a part B2 and its remain part B3 of the boundary we have a Signorini type condition.

We study the numerical analysis and the corresponding error bounds between the continuous and the discrete condition in order to obtain a two-phase Stefan-Signorini problem (that is, the solution is a function of non-constant sign in the whole domain) through the numerical analysis of the variational inequalities by using the finite element method.

Alfred Wagner

Institüt für Mathematik - Koëln, Germany

THE INSTATIONARY BERNOULLI PROBLEM

We consider an inviscid, incompressible and irrotational fluid in a bounded two dimensional domain $\Omega \times [0, T]$. Imbedded in the fluid (described by the potential u) we imagine a gas bubble of prescribed volume and pressure. The interface gas - fluid is assumed to be of constant pressure i.e. we do not include surface tension. The shape of the bubble is described as a level set of a function to be determined.

We prove the existence of a global generalised solution in classical spaces using the technique of generalised sub - and supersolutions. The solution may not be unique due to possible fattening effects of the interface.

A. Yu. Zubarev

Ural State University, Russia

CORRUGATION OF FERRONEMATIC FREE SURFACE

Ferronematics are colloidal suspensions of ferromagnetic particles in a liquid crystal. Unusual and outlooking for high technologies properties of these compounds attract considerable attention of researchers. In well-known papers the behaviour of ferronematics in the volumes with the fixed boundaries has been studied. The aim of this study is an analysis of the behaviour of ferronematic free boundary in magnetic field and in its absence. Two types of ferronematics have been considered, the first one being compensated (this means that magnetization in the absence of the field equals zero) and the second one being uncompensated (with the magnetization being constant in its absolute value). The situations have been studied when ferronematic director is normal and parallel to its boundary. The critical values of normal and tangential field above which the plate surface becomes unstable has been derived. In the supercritical fields the hexagonal or quadratic lattice of valleys and ridges is developed; the growth of deformations is limited by capillary forces and Frank tensions.

If the field applied is normal to ferronematic director and exceeds some critical value, then a deformation of a free surface reveals in conjunction with the space division of ferroparticles the moments of which are directed opposite to each other. In the most real cases the corrugation of a free surface is a soft phase transition;however, in certain cases it may occur as a rigid transition. It should be noted that the deformations of paramagnetic or dielectric liquids free surface in static tangential field has not been yet discussed in literature.

If magnetization of uncompensated (ferromagnetic) ferronematics is normal to its surface and exceeds critical value, then a free surface of such system becomes corrugative and hexagonal lattice of valleys and ridges occurs on this surface even in the absence of the field. The critical values of external field and ferronematic magnetization have been determined from the balance of capillary, magnetic and gravitational forces as well as Frank tensions in a liquid crystal matrix. The amplitude of the free surface corrugation is calculated as well as its period and the type of the occuring lattice. Depending on the system parameters, the corrugation of ferromagnetic liquid crystal free surface may appear as a soft or rigid phase transition.

POSTERS

Toyohiko Aiki

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Large-time behavior of two-phase Stefan problems for nonlinear parabolic equations

We consider the following two-phase Stefan problems: Find a function $u = u(t,x)$ on $Q(T) = (0,T) \times (0,1)$, $0 < T < \infty$, and a curve $x = \ell(t)$, $0 < \ell < 1$ on $[0,T]$ satisfying that

$$\rho(u)_t - a(u_x)_x + \xi + g(u) = \begin{cases} f_0 & \text{in } Q_\ell^{(0)}(T), \\ f_1 & \text{in } Q_\ell^{(1)}(T), \end{cases}$$

$$\xi(t,x) \in \beta(u(t,x)) \quad \text{for a.e. } (t,x) \in Q(T),$$

$$Q_\ell^{(0)}(T) = \{(t,x); 0 < t < T, 0 < x < \ell(t)\},$$

$$Q_\ell^{(1)}(T) = \{(t,x); 0 < t < T, \ell(t) < x < 1\},$$

$$u(t,\ell(t)) = 0 \quad \text{for } 0 \le t \le T,$$

$$\ell'(t) = -a(u_x)(t,\ell(t)-) + a(u_x)(t,\ell(t)+) \text{ for a.e. } t \in [0,T],$$

$$a(u_x)(t,0+) \in \partial b_0^t(u(t,0)) \quad \text{for a.e. } t \in [0,T],$$

$$-a(u_x)(t,1-) \in \partial b_1^t(u(t,1)) \quad \text{for a.e. } t \in [0,T],$$

$$u(0,x) = u_0(x) \quad \text{for } x \in [0,1],$$

$$\ell(0) = \ell_0,$$

where $\rho : R \to R$ and $a : R \to R$ are continuous increasing functions; β is a maximal monotone graph on R; $g : R \to R$ is a Lipschitz continuous function; $f_i (i = 0,1)$ is a given function on $(0,\infty) \times (0,1)$; $b_i^t (i = 0,1)$ is a proper l.s.c. convex function on R for each $t \ge 0$ and ∂b_i^t denotes its subdifferential in R; u_0 is a given initial function and ℓ_0 is a number with $0 < \ell_0 < 1$.

In this paper we discuss global existence in time of solution, global attractor and ω-limit set (cf. [1]). Recently, Ito, Kenmochi and Yamazaki [2] obtained the interesting results concerned with the attractor for non-autonoumous problems for the evolution equations in a Hilbert space.

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Asymptotic behavior of solutions to phase field equations with constraints under nonlinear dynamic boundary conditions

We consider a nonlinear system of the following form: Find functions $u : [0,+\infty) \to L^2(\Omega), w : [0,+\infty) \to L^2(\Omega)$ and $v : [0,+\infty) \to L^2(\Gamma)$ satisfying that

$$u_t + w_t - \Delta u = f(t,x) \quad \text{in } Q := (0,+\infty) \times \Omega, \tag{1}$$

$$\nu w_t - \kappa \bigtriangleup w + \beta(w) + g(w) \ni u \quad \text{in } Q, \tag{2}$$

$$u = v \quad \text{a.e. on } \Sigma := (0, +\infty) \times \Gamma, \tag{3}$$

$$\frac{\partial u}{\partial n} + c\frac{\partial v}{\partial t} + h(v) = 0 \quad \text{on } \Sigma, \tag{4}$$

$$\frac{\partial w}{\partial n} = 0 \quad \text{on } \Sigma, \tag{5}$$

$$u(0, x) = u_0(x), \ w(0, x) = w_0(x) \quad \text{for } x \in \Omega, \tag{6}$$

$$v(0, x) = v_0(x) \quad \text{for } x \in \Gamma, \tag{7}$$

where Ω is a bounded domain in R^N with smooth boundary Γ; ν, κ and c are positive constants; β is a maximal monotone graph in $R \times R$; $g : R \to R$ and $h : R \to R$ are Lipshitz continuous functions; f is a given function on Q; u_0, w_0 and v_0 are initial functions. $\frac{\partial}{\partial n}$ denotes the outward normal derivative on Γ. We denote by $\mathrm{CP}{=}\mathrm{CP}(u_0, w_0, v_0)$ the above system (1)$\sim$(7).

The boundary condition (4) for the unknown function u is called the nonlinear dynamic boundary condition, which includes the time derivative of the unknown function on the fixed boundary and is regard as a mathematical model for a feedback control device on the fixed boundary.

In the present talk we discuss the existence and uniqueness of the solution to CP, noting that the condition, $c > 0$, is required in order to prove the uniqueness by using the usual method for the evolution equation on the dual space of $H^1(\Omega)$.

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NUMERICAL ANALYSIS OF A CAVITATION FREE BOUNDARY PROBLEM WITH FLUX IMPOSED BOUNDARY CONDITIONS

In this work, an upwinding semidiscretized scheme based on characteristics method is analyzed when it is applied to a free boundary problem. The free boundary problem is issued from a cavitation model in lubrication and it involves especific flux imposed boundary conditions. The boundary conditions correspond to an axially lubricated journal bearing device and the mathematical analysis is the subject of the previous work. In this paper we prove that the specific semidiscretization method leads to a well posed family of semidiscretized problems that converges to the departure continuous problem.

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GLOBAL IN TIME VERSUS BLOWING UP SOLUTIONS TO CHEMOTAXIS SYSTEMS WITH FUNCTIONAL COEFFICIENTS

We study global in time solvability for initial data of arbitrary size to some simplified versions of the Keller–Segel model of chemotaxis

$$(*) \qquad \begin{aligned} u_t &= \nabla \cdot (\nabla u - u \nabla \phi(v)), \\ \varepsilon v_t &= \Delta v - v + u. \end{aligned}$$

This parabolic ($\varepsilon = 1$) or parabolic-elliptic ($\varepsilon = 0$) system is considered in a bounded smooth domain $\Omega \subset \mathbb{R}^N$ under the homogeneous Neumann conditions

$$\frac{\partial u}{\partial \nu} = \frac{\partial v}{\partial \nu} = 0 \ \text{ on } \ \partial\Omega \times (0, \infty),$$

and suitable initial conditions. The system ($*$) is a particular example of systems describing the movement of microorganisms sensitive to the gradient of a chemical secreted by themselves. Here $u = u(x,t) \geq 0$ and $v = v(x,t) \geq 0$ represent the densities of the population and of the chemical, $x \in \Omega$, $t \geq 0$. The function ϕ is called the *sensitivity* function, so that $\nabla\phi(v)$ is the velocity of the chemotactic movement.

Recently, much attention has been attracted by the basic model with $\phi(v) = v$ and $\varepsilon = 0$. In particular, the questions of global in time continuation of solutions with small initial data, versus finite time blow-up of large ones have been considered. Radially symmetric solutions that explode after a concentration of mass have been constructed by M. A. Herrero and J. J. L. Velázquez. Numerical evidence of blow-up at the boundary has been reported by K. Gajewski and K. Zacharias. Quantitative conditions (in terms of concentration of initial data measured by e.g. their Morrey space norms) have been given by the author for a related model of gravitational interaction of particles. An extensive list of problems under current study include characterization of blowing up solutions, a description of free boundary of the region of spreading of u and a comparison of chemotaxis systems with different sensitivity functions ϕ.

Here we generalize some results of T. Nagai and T. Senba who considered radially symmetric solutions of the system ($*$), $\varepsilon = 0$, in balls, give extensions to the nonradial case, and an extension to a parabolic system of chemotaxis with $\varepsilon = 1$.

T. Nagai and T. Senba studied the sensitivity functions $\phi(v) = \chi \log v$, $\phi(v) = \chi v^p$, with the constants $\chi > 0$ and $0 < p < 1$. Their results included (for the both functions ϕ) the global in time existence of radial solutions in the disk in the plane ($N = 2$), and for $\phi(v) = \chi \log v$ with $\chi < 2/(N-2)$ — in the balls in $\mathbb{R}^N$, $N \geq 3$. Moreover, for $N \geq 3$ and either logarithmic ϕ with $\chi > 2N/(N-2)$ or $\phi(v) = \chi v^p$, there exist radial solutions that blow up in a finite time. In certain sense, $\phi(v) = \chi \log v$ is a critical nonlinearity for the existence of global solutions to ($*$).

THEOREM 1. *Suppose that $N = 2$ and the sensitivity function ϕ is strictly sublinear: $0 < \phi'(v) \to 0$ when $v \to \infty$, as well as $v\phi'(v)$ is increasing. Then each radially symmetric solution to ($*$) with $\varepsilon = 0$ and a bounded density $u(x,0)$ can be continued to the global in time one.*

THEOREM 2. *If $N \geq 2$, $\phi(v) = \chi \log v$ and $\chi < 2/N$, then the system ($*$) with $\varepsilon = 0$ and arbitrary initial data $u(x,0) \in L^q(\Omega)$ (for some $q > N/2$) has global in time solutions.*

THEOREM 3. *If $N = 2$ and $\phi(v) = \chi \log v$ with $\chi \leq 1$, then for all initial data the system ($*$) with $\varepsilon = 1$, $u(x,0) \in L^2$ and $\nabla v(x,0) \in L^2$ has global in time solutions.*

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DETERMINATION OF UNKNOWN THERMAL COEFFICIENTS THROUGH PARABOLIC FREE BOUNDARY PROBLEMS

We determine unknown thermal coefficients of a semi-infinite material through a phase-change process with an overspecified condition on the fixed faceby considering two parabolic free boundary problems. In both problems we obtain formulae for the unknown thermal coefficients and, the necessary and sufficient conditions for the existence of an explicit solution of the corresponding

free-moving boundary problem. In the first part we consider a material of Storm's type with non-constant thermal coefficient. Here, we study a free boundary and a moving boundary problem for the heat equation (Stefan-like problem) for 9 cases. In the second part, we consider a free boundary problem for a non-linear heat conduction equation with a convective term for 14 cases.

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MODELLING AND CONTROL OF MARTENSITIC PHASE TRANSITIONS IN SHAPE MEMORY ALLOYS

Shape memory alloys (SMA; e.g. CuZnAl, NiTi, ...) have received increasing attention in recent years partly due to the wide range of their technical applic ability in actuators and adaptive structures. They show a noticeable change in their mec hanical behaviour, i.e. stress–strain relation in different temperature ranges: elastic at high tempera tures and pseudo– or quasiplastic at low temperatures. An alloy can be permanently de formed to up to 10% without fracture and still it recovers its old shape under heating or co oling. This is due to first order structural phase transitions between different equilibrium configurations of the metallic l attice, named austenite and martensite. Austenite is the undeformed crystal lattice which is stable at high temperatures. By deforming the lattice, one obtains 24 crystallographically equivalent variants. These martensite variants prevail at low temperatures.

In order to exploit these phenomena for industrial applications a satisfactory m odel is needed. We develop a thermodynamically consistent one–dimensional Landau–Ginzburg model which is able to describe all relevant effects [1,2]. For instance, we investigate a so-called deformation-driven experiment (DDE): a thin rod of a SMA is fixed on one side and pushed and pulled on the other side in the course of time by an elongation m. To demonstrate the validity of our model we will compare the numerical results to experiments performed by I. Müller and his co –workers [3,4]. Summarizing, for a DDE we have the following system ($\Omega := (0,l)$, $Q := \Omega \times (0;T)$):

$$\rho\, u_{tt} - (\gamma\,(\theta-\theta_1)\,u_x - \beta\, u_x^3 + \alpha\, u_x^5)_x + \delta\, u_{xxxx} = 0, \quad \text{in} \quad Q, \tag{1}$$

$$c_e\,\theta_t - \kappa\,\theta_{xx} - \gamma\,\theta\, u_x\, u_{xt} = g(x,t), \quad \text{in} \quad Q, \tag{2}$$

$$u(0,t) = u_{xx}(0,t) = u_{xx}(l,t) = 0, \quad u(l,t) = m(t),$$
$$\theta_x(0,t) = 0, \quad -\kappa\,\theta_x(l,t) = \bar{\kappa}\,(\theta(l,t) - \theta_\Gamma(t)), \quad \forall t \in [0,T],$$
$$u(x,0) = u_0(x), \quad u_t(x,0) = u_1(x), \quad \theta(x,0) = \theta_0(x), \quad \forall x \in \overline{\Omega}.$$

(1) and (2) represent the balance laws of momentum and energy, respectively. It denotes: ρ – mass density, u – displacement, θ – absolute te mperature, u_x – strain, c_e – specific heat, κ – heat conductivity, g – density of heat sources or sinks, l – length of the rod, $\bar{\kappa}$ – heat exchange coefficient, θ_Γ – outside temperat ure. The couple stress leads to the Ginzburg–term $\delta \cdot u_{xxxx}$, and α, β, γ, and δ are material constants to be determined for each specimen.

We discuss existence and uniqueness results for (1) (see [5]), we outline the determination of the physical parameters, and we present our nume rical simulations [2]. The main features are:

1) Displacement, strain and temperature evolution show the experimentally observed phenomena: phase transitions between variants o f martensite or between austenite and one variant, respectively, changes of temperatures and latent heats, nucleation processes, propagation of p hase boundaries.
2) Size and shape of our simulated hysteresis loops are independent of dicret ization. The coefficient of the term modelling the interfacial energy, i.e. δ, det ermines the size of the loop and leads to quantitative agreement with experiments. This corresponds to the model of [3].
3) The model is even capable of producing interior hysteresis loops which are also found in the experiments.

In order to control the system, we will consider a weak formulation of (1) ($\mathcal{WF}$). The aim is to achieve, possibly isothermally, a prescribed distribution of the phases. Therefore, it is natural to consider a cost functional involving the order parameter $\varepsilon = u_x$ and θ (or rather, the stress $\sigma = \gamma(\theta - \theta_1)u_x - \beta u_x^3 + \alpha u_x^5$), as well as the natural control variables m, θ_Γ, and g. For instance, one could investigate the case when

$$J(m, g, \theta_\Gamma) = \alpha_1 \| \sigma - \overline{\sigma} \|^2_{L^2(Q)} + \alpha_2 \| \theta - \overline{\theta} \|^2_{L^2(Q)} + \alpha_3 \|\ddot{m}\|^2_{L^2(0,T)} + \alpha_4 \| g \|^2_{L^2(Q)} + \alpha_5 \| \theta_\Gamma \|^2_{L^2(0,T)},$$

where α_i, $i = 1, \ldots, 5$, are non–negative constants, and where $\overline{\sigma}$ and $\overline{\theta}$ denote the desired stress and temperature distribution duri ng the evolution of the process, respectively. Then our control problem reads

(CP) Minimize $J(m, g, \theta_\Gamma)$, subject to $\mathcal{WF}$ and $(m, g, \theta_\Gamma) \in \mathcal{U}_{ad}$.

Here, $\mathcal{U}_{ad}$ denotes the set of admissible controls, and is some nonempty, convex, bounded, and closed set.

We prove the Fréchet differentiability of the solution operator and we derive the necessary conditions of optimality [5]. Of course, also other cost functionals are conceiv able in actual applications.

Furthermore, investigating a more realistic case, we include the following state constraints for stress and temperature [6], i.e.

$(\theta, \sigma) \in \mathcal{C}$, with $\mathcal{C}$ given by

$$\mathcal{C} := \left\{ (\theta, \sigma) \in C(\overline{Q}) \times C(\overline{Q}) \middle| c_1 \le \theta(x,t) \le c_2, \;\; c_3 \le \sigma(x,t) \le c_4, \;\; \forall\, (x,t) \in \overline{Q} \right\}.$$

Again, the necessary optimality conditions are given.

As an application we are going to dampen out the vibrations of a thin rod by hea ting and cooling. We want to emphasize the fact that all these kind of problems can be regarded as subproblems for actuators and adaptive structures.

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Pointwise and viscosity solutions for the limit of a two phase parabolic singular perturbation problem

In [CLW1], [CLW2] we deal with the problem of studying the uniform properties and the limit, as $\varepsilon \to 0$, of solutions $u^\varepsilon(x,t)$ of the equation

$$(P_\varepsilon) \qquad \Delta u^\varepsilon - u^\varepsilon_t = \beta_\varepsilon(u^\varepsilon),$$

where $\varepsilon > 0$, $\beta_\varepsilon \geq 0$, $\beta_\varepsilon(s) = \frac{1}{\varepsilon}\beta(\frac{s}{\varepsilon})$, support β=[0,1] and $\int \beta(s)\,ds = M$.

We consider a family u^ε of solutions to P_ε in a domain $\mathcal{D} \subset \mathbb{R}^N 1$ which are uniformly bounded in L^∞ norm in $\mathcal{D}$. We get uniform estimates that allow us to pass to the limit as $\varepsilon \to 0$, we find properties of the limit function u in general situations, and we prove that u is a solution to the free boundary problem

$$(P) \qquad \begin{array}{l} \Delta u - u_t = 0 \qquad \text{in } \mathcal{D} \setminus \partial\{u > 0\} \\ u = 0 \ , \ \ (u^+_\nu)^2 - (u^-_\nu)^2 = 2M \quad \text{on } \mathcal{D} \cap \partial\{u > 0\}. \end{array}$$

Here ν is the inward unit spatial normal to the free boundary $\mathcal{D} \cap \partial\{u > 0\}$, $u^+ = \max(u,0)$ and $u^- = \max(-u,0)$.

On one hand, we prove that every limit function u is a solution to P in a very weak sense. Moreover, at any free boundary point (x_0,t_0) there holds that if $\limsup_{(x,t)\to(x_0,t_0)} |\nabla u^-| \leq \gamma$, then $\limsup_{(x,t)\to(x_0,t_0)} |\nabla u^+| \leq \sqrt{2M+\gamma^2}$.

On the other hand, we prove that, in the two phase case, the free boundary condition is satisfied in a pointwise sense at any "regular" free boundary point (x_0,t_0). We point out that we only make assumptions on u at (x_0,t_0). More precisely, if $(x_0,t_0) \in \mathcal{D} \cap \partial\{u > 0\}$ is such that there exists an inward unit spatial normal to the free boundary $\mathcal{D} \cap \partial\{u > 0\}$ in a parabolic measure theoretic sense and such that $\{u \equiv 0\}$ has zero "parabolic density" at (x_0,t_0) then, there exist $\alpha > 0, \gamma \geq 0$ such that

$$u(x,t) = \alpha\langle x - x_0, \nu\rangle^+ - \gamma\langle x - x_0, \nu\rangle^- + o(|x - x_0| + |t - t_0|^{\frac{1}{2}})$$

with $\alpha^2 - \gamma^2 = 2M$. Here ν is the inward unit spatial normal to the free boundary at (x_0,t_0).

Finally we prove that, when $\{u \equiv 0\}^\circ = \emptyset$, the limit function u is a viscosity solution to the free boundary problem P. By a viscosity solution, we mean a weak solution of the free boundary problem in the sense that it is a continuous function for which comparison principles with classical supersolutions and subsolutions hold.

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LOCALIZATION PROPERTIES FOR A QUASILINEAR DEGENERATED SYSTEM IN SEMICONDUCTORS THEORY

In this joint work with **J.I. Díaz** and **A. Jüngel**, we consider the following problem: let u (electrons density), v (holes density) and w (electric potential) be any solution of the system

$$\left\{ \begin{array}{c} u_t - div\,(\nabla\varphi(u) - b(u)\nabla w) = F(u,v) \\ v_t - div\,(\nabla\varphi(v) + b(v)\nabla w) = F(u,v) \\ -\Delta w = v - u + C. \end{array} \right\}.$$

Find conditions on φ and F that give rise to the existence of free boundaries. We use a local energy method combined with the consideration of the characteristics defined by ∇w to get the following results:If $\varphi(s) := s^m$, with $m > 1$ then if $F(s,\sigma)$ satisfies

$[F(s,\sigma)(s^m + \sigma^m) \leq c\left(s^{m+1} + \sigma^{m+1}\right),]$ then u, v have the property of *finite speed of propagation.* Moreover, if $F(s,\sigma)$ satisfies $[-F(s,\sigma)(s^m + \sigma^m) \geq c\left(s^{m(p+1)} + \sigma^{m(p+1)}\right),$ with $0 < p < \frac{2}{m} - 1]$ and the global energy is bounded then, even if the initial data are strictly positive, a *dead core* arises.

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DOMAIN DECOMPOSITION METHODS FOR THE STEFAN PROBLEM IN NON HOMOGENEUS MEDIA: SOME INDUSTRIAL APPLICATIONS

Nonoverlapping domain decomposition methods are very adecuated to solve problems where certain "difficult" phenomena occurs only in a specific subdomain, whereas in the rest of the domain a standar equation applies. This can be the case of the Stefan problem in non homogeneus media. A lineal model problem is formulated as an optimization problem with constrains, and so reduced to the localization of a saddle point of a Lagrangian or Augmented Lagrangian funtionals. This idea is due to Glowinski and Le Tallec (1990) but is fully developed in this work. We present a wide family of nonoverlapping decomposition methods and study their convergence properties for the linear case. We also present some numerical results for the Stefan problem in non homogeneus media related to the following industrial applications: a)The aluminium solidification in a cast. b)The deodorization of vegetable oils which involves condensation and freezing of a multicomponent vapor mixture inside shell and tube heat exchanger.

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GROWTH OF THE VISUAL IMAGE IN ELECTROPHOTOGRAPHY

Friedman and Velazquez [1] have modelled the development of the visual

image in electrophotography as a time-dependent, free boundary problem. They proved results on existence and uniqueness of a solution to a two-dimensional version of the problem. We present a numerical solution to the same version of the problem and find the shape of the toner layer (i. e.

the visual image) as a function of time. At the initial instant, an exact expression can be derived for the electric field and, hence, for the growth rate of the toner layer. We use that expression to start the computation. We also derive an integral equation formulation of the free boundary problem, and use that formulation to find the shape of the toner layer at later times.

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TWO LIMITING REGIMES OF THE DIRECTIONAL SOLIDIFICATION OF BINARY MELTS WITH THE MUSHY REGION

The concentrational supercooling of a binary melt before crystallization front results in the origination of mushy region. An analysis of the structure of the mushy region, which includes the liquid, solid particles, and dendrites extending from the bulk solid surface, is suggested. The analytical solution of the quasi-equilibrium two-phase zone problem is obtained on the basis of the small parameter method; the temperature, concentration of solute inside the region, and solid phase volume fraction are found. If the zone thickness is much smaller than the pertinent linear scales of the process under consideration, the zone itself may be substituted by a discontinuity interface. A new formulation of the corresponding mathematical problem with an unknown boundary is suggested, which modifies the known Stefan problem by means of replacing the mass balance condition at a smooth interface with a new one stemming from the analysis of the zone structure and enables to derive physically transparent conclusions.

When heat supply to a binary melt at its outer boundary is absent, macrokinetics of solidification are governed by progressive cooling of the melt rather than by withdrawal of an admixture. Because the former process is much faster than the latter one, a metastable region of concentrational supercooling arises and has time to expand before the ingot surface noticeably changes its position. This leads to bulk crystallization within such a region prior to solidification at the surface. If crystalline grains are forming and growing rapidly enough, the two-phase region is practically at the state of thermodynamical equilibrium. A simple self-similar solution of releavant equations is obtained which approximately decsribes unidirectional process.

Akio Ito

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STABILITY FOR SOLUTIONS OF ALLEN-CAHN EQUATION

We consider the asymptotic behaviour of Allen-Cahn equation with constraint in one-dimensional space:

$$w_t - (|w_x|^{p-2}w_x)_x + \xi + g(w) = 0 \quad \text{in } Q,$$

$$\xi \in \partial I_{[\sigma_*,\sigma^*]}(w) \quad \text{in } Q,$$

subject to initial and boundary conditions;

$$w_x(t, \pm L) = 0 \quad \text{for } t > 0,$$

$$w(0,x) = w_0(x) \quad \text{for any } x \in (-L, L).$$

Here $L > 0$ is a positive number; p is a constant with $2 \leq p < +\infty$; $g(w)$ is a smooth function of w; $\partial I_{[\sigma_*,\sigma^*]}$ is the subdifferential of the indicator function $I_{[\sigma_*,\sigma^*]}$ of the compact interval $[\sigma_*,\sigma^*] \subset R$.

For this model, we consider the following second-order elliptic problem as steady-state problem:

$$-(|w_x^\infty|^{p-2}w_x^\infty)_x + \xi^\infty + g(w^\infty) = 0 \quad \text{in } (-L,L),$$

$$\xi^\infty \in \partial I_{[\sigma_*,\sigma^*]}(w^\infty) \quad \text{in } (-L,L),$$

$$w_x^\infty(\pm L) = 0.$$

In this talk, we discuss:

(i) structure of the set of stationary solutions w^∞.

(ii) convergence of $w(t)$ to a stationary solution w^∞ as $t \to +\infty$.

(iii) stability of stationary solutions w^∞.

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VISCOSITY SOLUTIONS AND REGULARITY OF THE FREE BOUNDARY FOR THE LIMIT OF AN ELLIPTIC TWO PHASE SINGULAR PERTURBATION PROBLEM

In this work we are concerned with the following problem, of interest in combustion: Study the limit as $\varepsilon \to 0$, of solutions $u^\varepsilon(x)$ to the equation

$$(E_\varepsilon) \qquad \Delta u^\varepsilon = \beta_\varepsilon(u^\varepsilon),$$

where $\varepsilon > 0$, $\beta_\varepsilon(s) = \frac{1}{\varepsilon}\beta(\frac{s}{\varepsilon})$, $\beta > 0$ in $(0,1)$, $\beta \equiv 0$ otherwise, and $\int \beta(s)ds = M$.

Given a family u^ε of uniformly bounded solutions to E_ε in a domain $\Omega \subset \mathbb{R}^N$ one expects that the limit function u be a solution to the free boundary problem:

$$(E) \qquad \begin{aligned} &\Delta u = 0 \qquad \text{in } \Omega \setminus \partial\{u>0\}, \\ &u = 0,\ (u_\nu^+)^2 - (u_\nu^-)^2 = 2M \qquad \text{on } \Omega \cap \partial\{u>0\}, \end{aligned}$$

where ν is the inward unit normal to the free boundary $\Omega \cap \partial\{u > 0\}$. In fact, in this work we prove the following result s for the limit function u:

1) Let $x_0 \in \partial\{u > 0\}$ be such that there exists an inward unit normal ν to $\partial\{u > 0\}$ at x_0 in the measure theoretic sense. If either the set $\{u < 0\}$ has positive density or u^+ is nondegenerate at x_0 then u satisfies the free boundary condition at x_0. I.e., there exist both u_ν^+ and u_ν^-, and $(u_\nu^+)^2 - (u_\nu^-)^2 = 2M$ at x_0.

2) Under suitable assumptions, u is a solution to E in a viscosity sense. By a viscosity solution we mean a continuous function, harmonic away from the free boundary, which satisfies the free boundary condition in terms of proper asymptotic developments. In particular, u is a viscosity solution if $\{u \equiv 0\}^\circ = \emptyset$ or if u^+ is

in terms of proper asymptotic developments. In particular, u is a viscosity solution if $\{u \equiv 0\}^\circ = \emptyset$ or if u^+ is nondegenerate on $\partial\{u > 0\}$.

3) We prove, in addition, the following regularity results:

3.i) Under suitable assumptions on the limit function u, there is a subset of the free boundary $\partial\{u > 0\}$ which is locally a $C^{1,\alpha}$ surface. Thus, on this portion of $\partial\{u > 0\}$, the free boundary condition is satisfied in the classical sense. Moreover, this smooth subset is open and dense in

$\partial\{u > 0\}$ and the remainder of the free boundary has $(N-1)$-dimensional Hausdorff measure zero.

3.ii) If u^- is locally uniformly nondegenerate on $\partial\{u > 0\}$, then the free boundary is locally a $C^{1,\alpha}$ surface. Hence, there are no singularities and u is a classical solution to E. We point out that the loca l uniform nondegeneracy of u^- on $\partial\{u > 0\}$ is a necessary condition for the regularity of the free boundary in the strictly two phase case, even if we only require the free boundary to be locally Lipschitz continuous.

We finally want to remark that there are limit functions u which do not satisfy the free boundary condition in the classical sense on any portion of $\partial\{u > 0\}$ (for instance, $u = \alpha x_1^+ + \alpha x_1^-$ with $0 < \alpha < \sqrt{2M}$).

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UNIQUENESS OF SOLUTION TO A FREE BOUNDARY PROBLEM FROM COMBUSTION

We report the results in [LVW] where we investigate the uniqueness and agreement between different kinds of solutions for a free boundary problem in heat propagation that in classical terms is formulated as follows: to find a continuous function $u(x,t) \geq 0$, defined in a domain $\mathcal{D} \subset \mathbb{R}^N \times (0,T)$ and such that

$$u_t = \Delta u \quad \text{in} \quad \mathcal{D} \cap \{u > 0\}.$$

Besides we assume that the interior boundary of the positivity set, $\mathcal{D} \cap \partial\{u > 0\}$, so-called free boundary, is a regular hypersurface on which the following conditions are satisfied

$$u = 0, \quad -\partial u/\partial\nu = C$$

(ν the outward unit spatial normal). In addition, initial data are specified , as well as either Dirichlet or Neumann data on the external boundary of $\mathcal{D} \cap \{u > 0\}$. This problem arises in combustion theory as a limit situation in the propagation of premixed flames (high activation energy limit).

This free boundary problem admits classical solutions only for good data in the small. Several generalized concepts of solution are proposed, among them the concepts of limit solution and viscosity solution. We investigate conditions under which the three concepts agree and produce a unique solution.

A limit solution is a function which is obtained as the limit (as $\varepsilon \to 0$) of solutions u^ε to

$$\Delta u^\varepsilon - u_t^\varepsilon = \beta_\varepsilon(u^\varepsilon)$$

where $0 \leq \beta_\varepsilon \leq \frac{K}{\varepsilon}$, supp $\beta_\varepsilon = [0,\varepsilon]$ and $\int \beta_\varepsilon(s)\,ds = \frac{C^2}{2}$.

A viscosity solution is a continuous function which satisfies comparison principles with classical supersolutions and subsolutions to the free boundary problem.

Our results can be summarized as saying that, under suitable assumptions on the domain, the reaction function β_ε and on the initial and boundary data, *if a classical solution to the free boundary problem exists in a certain time int erval, then it is at the same time the unique classical solution, the unique limit solution and also the unique viscosity solution in that time interval.*

For related works see [CV], [CLW], [GHV].

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ON A NON-UNIFORMLY ELLIPTIC SYSTEM BOUNDARY VALUE PROBLEM

Linear boundary values problems for a degenerate nonlinear system of the Lavrentiev type equations, in the case when the index corresponding to the boundary values problems are negative, has been investigated. Making use of functiontheoretic methods in the theory of partial differential equations, existence, uniqueness and stability of the solution of boundary problems in complex form, for non-uniformly ellipticity of the equation in the Sobolev space are introduced.

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SOLUTION OF NONLINEARLY CURVATURE DRIVEN EVOLUTION OF PLANE CURVES

We investigate the evolution of closed smooth planar curves $\Gamma : R/Z \to R^2$ satisfying the geometrical equation

$$v = \beta(k) \tag{1}$$

where v is the normal velocity of evolving curve and β is a smooth function of the curvature k. As a typical example one can consider a function $\beta(k) = k^m$ where $m > 0$. Geometrical equations of type (1) are used for describing various phenomena in physics, material sciences, computer vision, robotics and artificial intelligence. There are two main fields in which the evolution of plane curves play an important role:

a) the multiscale analysis of images and shapes closely related to signal smoothing, edge detection, segmentation and image representation;

b) the Stefan problem with surface tension and related interface motion models.

We suggest a computational method for solving (1). The aim is to represent the equation by a so-called intrinsic heat equation

$$\frac{\partial x}{\partial t} = \frac{\partial^2 x}{\partial s_*^2} \tag{2}$$

where s_* is an appropriate parametrization of a curve corresponding to the function β. Then, we make use of the 'Eulerian transformation' of the intrinsic heat equation into a degenerate evolutionary partial differential equation with spatial variable being independent on time and varying on a fixed interval. This equation is a generalization of the corresponding one studied by Dziuk for classical curve shortening flow problem. We prove some a-priori estimates of a smooth solution which, in particular, imply the *curve shortening* property of the governing equation. The same property is proven for time semidiscretization. The proposed numerical scheme is carefully tested by various examples of the nonlinear curvature driven evolution (1). Numerical results are compared with the exact selfsimilar solutions and also with previous results obtained by a conceptually different method introduced by Mikula and Kačur (which can be used only in convex case). We apply the method for computing the evolution of nonconvex, selfintersecting and nonrectifiable curves as well.

This is a joint work with **Daniel Ševčovič**.

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THE DESTABILISING EFFECT OF SURFACE NON-UNIFORMITIES ON THIN FLUID FILMS FLOWING DOWN AN INCLINATION

In recent studies it could be shown that linear stability analysis for a thin film flowing down an inclined plane predicts stabilisation below a certain critical inclination angle α^* depending on the static contact angle θ_S. Experiments, however, have shown finger formation for inclination angles significantly below α^* in situations where θ_S is small.

Since every realistic plane surface exhibits non-uniformities at a microscopic scale, the possibility that these may serve as an additional source of instability is considered in this work.

We model the non-uniformities as a perturbation of the slip length, a parameter which enters in a singular fashion into the governing lubrication equation for the thin film.

On the basis of our model we present numerical evidence that these perturbations are amplified to macroscopic changes in the fluid profile and the contact line, supporting the proposition that this effect should play an important role explaining the mechanism of destabilisation for the regime of small inclination and contact angles.

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OPTIMAL CONTROL APPROACH FOR THE FLUID-STRUCTURE INTERACTION PROBLEMS

A fluid-structure interaction problem is studied. We are interested by the displacement of the structure and by the velocity and the pressure of the fluid.

The contact surface between fluid and structure is unknown a priori, therefore it is a **free boundary like problem.**

In the classical approaches, the fluid's and structure's equations are coupled via two boundary conditions: equality of the fluid's and structure's velocities at the contact surface (which is a Dirichlet like boundary condition) and equality of the fluid's and structure's forces at the contact surface (which is a Neumann like boundary condition).

In our approach, the Dirichlet like contact boundary condition will be relaxed and treated by the Least Squares Method.

We start with a guess for the contact forces. The displacement of the structure can be computed. We suppose that the fluid's domain is completely determined by the displacement of the structure. Knowing the actual domain of the fluid ant the contact forces, we can compute the velocity and the pressure of the fluid.

In this way, the Neumann like contact boundary condition is trivially accomplished.

The problem is to find the contact forces such that the Dirichlet like contact boundary condition holds. It's a exact controllability problem with Neumann like boundary control with Dirichlet like boundary observation. The control appears also in the coefficients of the fluid's equations.

In order to obtain some existence results, this exact controllability problem will be transformed in an optimal control problem using the Least Squares Method.

This mathematical model permits to solve numerically the coupled fluid-structure problem via partitioned procedures (i.e. in a decoupled way, more precisely the fluid's and the structure's equations are solved separately).

The aim of this paper is to present the existence of an optimal control for a fluid-beam interaction problem.

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Asymptotic convergence of the Stefan problem to Hele-Shaw

A Stefan problem is a mathematical model for describing the melting of a body of ice in contact with a region of water. In one-phase Stefan problems, the temperature of the ice is supposed to be maintained at $0°C$. Hence, the unknowns are the temperature distribution in the water and the shape of the interface (free boundary) between ice and water.

The Hele-Shaw problem is a model describing the movement of a viscous fluid confined in a narrow cell between two parallel plates. It has applications in the plastics industry (injection moulding) and in electromachining. It can also be considered as the zero specific heat limit of the Stefan problem, being in that sense a simplified version of that famous problem.

The aim of this work (detailed proofs are given in [2]) is to study the large-time behaviour of the solution to the initial-and-boundary-value problem in an exterior domain, Ω, for both models, and to show their relationship. We prescribe Dirichlet boundary data $g(x)$ on the compact set $\partial\Omega$, which are non-negative, non-trivial ($g \not\equiv 0$) and constant in time. For the Stefan problem we also specify initial data $\theta(x,0) = \theta_0(x)$, which are assumed to be nonnegative and compactly supported in Ω. For the Hele-Shaw problem we need only specify the initial support, which is bounded. The classical formulation of these problems in spacial dimension $n = 1$ is the following:
1-D Stefan problem. To find $\theta(x,t)$ and $r(t)$ such that

$$\begin{cases} \dfrac{\partial\theta}{\partial t} = \dfrac{\partial^2\theta}{\partial x^2} & 0 < x < r(t),\ t > 0, \\ \theta(x,0) = \theta_0(x) & 0 \le x \le b, \\ \theta(r(t),t) = 0 & t \ge 0, \\ \dfrac{\partial\theta}{\partial x}(r(t),t) = -Lr'(t) & t > 0, \\ \theta(0,t) = A & t \ge 0, \\ r(0) = b. \end{cases}$$

1-D Hele-Shaw problem. To find $p(x,t)$ and $r(t)$ such that

$$\begin{cases} \dfrac{\partial^2 p}{\partial x^2} = 0 & 0 < x < r(t),\ t > 0, \\ p(r(t),t) = 0 & t \ge 0, \\ \dfrac{\partial p}{\partial x}(r(t),t) = -Lr'(t) & t > 0, \\ p(0,t) = A & t \ge 0, \\ r(0) = b. \end{cases}$$

We will show that there is a marked difference in the asymptotic behaviour of these problems in one and several dimensions. Thus, though in one space dimension there is convergence to a stationary state in the $x/t^{1/2}$-scale for both problems, the asymptotic profile is not the same. The situation is different for $n > 1$. Indeed, there is no self-similar solution which explains simultaneously the large-time behaviour of the solutions and interfaces of our problems. The actual behaviour can be described using the ideas of *matched asymptotic expansions*. The positivity set expands to cover in finite time any compact subset of the exterior domain Ω and, in the usual (x,t) coordinates, the solutions of both problems stabilize to the solution of the exterior Dirichlet problem for the Laplacian. This is what we call the *near field* or *regular* limit. On the other hand, we prove that, after a scaling of the form

$$\begin{aligned} &y = x/t^{1/n}, && v = ut^{(n-2)/n} && \text{if } n \ge 3, \\ &y = x/\mathcal{R}(t), && v = u \log \mathcal{R}(t) && \text{if } n = 2, \end{aligned}$$

where $\mathcal{R}(t) \sim C(t/\log t)^{1/2}$ as $t \to \infty$, and u represents the temperature if we are studying the Stefan problem and the pressure if we are considering the Hele-Shaw flow, we have convergence in both problems to the same free boundary problem, which corresponds to a singular, radial, self-similar solution of the Hele-Shaw problem. This is what we call the *far field* or *singular* limit. This means that in the *far* region, close to the free boundary, Hele-Shaw represents the *reduced problem* for Stefan (in the sense of Prandtl's theory of boundary layers). In practice, the heat equation of the Stefan problem *loses* the inertial term θ_t as $t \to \infty$ because this term becomes lower order with respect to the main terms at the asymptotic level. It is thus a case of *asymptotic simplification*, a rather usual phenomenon in nonlinear parabolic equations. The analysis gives in particular the position of the free boundary, whose behaviour is given by

$$
\begin{aligned}
|x| &\sim Ct^{1/n} && \text{if } n \geq 3,\\
|x| &\sim C(t/\log t)^{1/2} && \text{if } n = 2.
\end{aligned}
$$

The *matching* between both regions amounts to adjusting the constant C in the far region.

A previous study of the asymptotic behaviour for the Stefan problem for $n \geq 3$ was done by Matano in [1]. He proves that any weak solution becomes eventually classical, that is, the interface between the ice and the water regions is sufficiently smooth for all large t. He also proves that the shape of the free boundary (that is, the interface) tends to a sphere of radius $(Ct)^{1/n}$ as $t \to \infty$ and calculates the constant. In addition to that, we establish the asymptotic developments for all $n \geq 2$. We also prove that the shape of the free boundary of the Hele-Shaw problem approaches a sphere, with precisely the same radius.

In conclusion, there is a strong difference between the one-dimensional and the multidimensional cases: for $n = 1$ solutions of the Stefan problem do not eventually become solutions of Hele-Shaw, while for $n \geq 2$ they do. There is a explicit selfimilar solution giving the asymptotic behaviour of Stefan when $n = 1$ which *is not* a solution of Hele-Shaw. For $n \geq 2$ we need two matched developments to explain the asymptotic behaviour of the solutions in both problems. For the record, the development in the region close to the free boundary is always self-similar. The influence of the compactly supported initial data disappears (to first order) in all cases.

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NON-ISOTHERMAL INJECTION MOULDING WITH RESIN CURE, PREFORM DEFORMABILITY, AND COUPLING BETWEEN INTERFACED DOMAINS

The paper deduces a three-dimensional model based on the theory of deformable porous media aimed at simulating some injection moulding processes used to fabricate composite materials. It works under non isothermal conditions, includes resin cure and allows the solid constituent in both the dry and the wet region to deform during infiltration. The model is simplified in the one-dimensional case by performing analytically the integrations of the mechanical equations in the uninfiltrated region. The remaining system of partial differential equations in the two interfaced and time-dependent domains is then posed with the proper interface and boundary conditions both in the case of given inflow velocity and in the case of given pressure cycle. After writing for numerical reasons the problem in a Lagrangian formulation fixed on the solid constituent domain decomposition techniques are used for the simulation.

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In this paper we consider an inverse problem of heat equation with free boundary condition. We identify necessary initial and boundary temperature to create a certain free boundary.

George G. Tsypkin

MOVING BOUNDARY PROBLEMS OF GEOTHERMAL RESERVOIR MODELING

Coupled processes of heat and multi-phase mass transfer with phase transitions in porous media are of practical interest in geophysical and engineering sciences. A large class of these problems are Stefan - like problems. A new mathematical models of water-ice-vapor phase transitions in frozen soils [1], gas hydrate dissociation in strata [2], and phase transition water-steam in geothermal reservoirs [3] have been proposed. These models are generalizations of the Stefan problem and take into account appearance of two unknown moving phase transitions boundaries and (or) extended phase transition zones ("mushy regions"). The models include equations of mass and energy conservation laws, the generalized Darcy law and thermodynamic relations. The formulation of each problem contains moving interface at which the saturation function suffers a discontinuity. An essential part of the mathematical description is the correct specification of boundary conditions. The conservation laws may be written in term of "jump" at phase transition fronts. We assume that all phase at the fronts are locally at a thermodynamic equilibrium. These relations constitute the complete system of boundary conditions at the moving interface. On the whole the problems are nonlinear. It is vital to emphasize that in all zones it is possible to apply linear approach and derive the self-similar analytical solutions for half space domain. The solutions in all regions can be expressed in terms of linear combinations of probability integrals.

By substituting these solutions into the boundary conditions at the moving fronts we obtain the systems of transcendental equations for unknown functions, which are solved numerically. For example, the problems of steam (steam-water mixture) production from geothermal reservoirs are considered. It is shown that the introduction of an evaporation surface separating the steam and water leads to superheating of the water in a zone ahead of the front. This contradiction is removed by introducing of an extended phase transition zone ("mushy region") between the single-phase zones. Water and steam in this region are locally at a thermodynamic equilibrium. The boiling front separating the water zone and the two-phase one is degenerated. In this case condition of continuity of water saturation function is satisfied. This function at the mixture-steam front varies discontinuously from the unknown value to zero.

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EXISTENCE OF PERIODIC SOLUTION FOR PHASE TRANSITIONS

We consider a solid-liquid phase transition model.

Let θ be the absolute temperature and w be the order parameter. Then our model is a couple of system of nonlinear second order parabolic PDEs (kinetic equations for internal energy and the order parameter) as follows:

$$[\theta + \lambda(w)]_t - \Delta\left(-\frac{1}{\theta}\right) + \mu\theta = f(t,x) \quad \text{in } Q := (0,+\infty)\times\Omega,$$

$$w_t - \kappa\Delta w + \beta(w) + g(w) + \frac{\lambda'(w)}{\theta} \ni 0 \quad \text{in } Q,$$

with boundary conditions

$$\frac{\partial}{\partial n}\left(-\frac{1}{\theta}\right) + n_0\left(-\frac{1}{\theta}\right) = h(t,x),\ \frac{\partial w}{\partial n} = 0 \quad \text{on } \Sigma := (0,+\infty)\times\Gamma,$$

and T_0-periodic conditions

$$\theta(0,\cdot) = \theta(T_0,\cdot), \quad w(0,\cdot) = w(T_0,\cdot) \quad \text{on } \Omega.$$

Here Ω is a bounded domain in $R^N (1 \le N \le 3)$ with smooth boundary $\Gamma := \partial\Omega$; $\beta(\cdot)$ is a maximal monotone graph in $R \times R$; $\lambda(\cdot)$ is a smooth convex function on R; $g(\cdot)$ is a smooth function on R; n_0, κ and μ are positive constants; T_0 is a positive constant and f, h are given data which are T_0-periodic in time.

In this talk, we show the existence of periodic solution of our model.

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THE SIMULATION OF INSTABILITIES FOR TWO-PHASE FLOW BETWEEN PARALLEL PLATES

The hydrodynamical instabilities are interesting phenomena, which are important for different technological processes. We investigate the instabilitie s of two-phase flow between two parallel plates. The mathematical model is based on well known approximation of the Hele-Shaw flow. We solve the problem with finite difference method. For describing of the moving free boundary it is convenient to use the level-set approach [C].

The level-set method needs values of the speed at grid-points nearby the interface. Because the viscosity and the pressure have jumps on the interface, it is an open problem, how to approximate the coupling conditions and calculate the speed. We compare two different approximations. One is based on the smoothed delta-function, the other approximation is based on ideas of the immersed interface method [B],[A]. The proposed methods allow the simulation of instabilities that lead to changes of the topology of phases. We investigate both methods and present numerical results.

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